UND-HEP-99-BIG01
January 1999

CP VIOLATION AND BEAUTY DECAYS – A CASE STUDY OF HIGH IMPACT, HIGH SENSITIVITY AND EVEN HIGH PRECISION PHYSICS [a]

I. I. BIGI

Physics Dept., University of Notre Dame du Lac, Notre Dame, IN 46556, USA
E-mail: bigi@undhep.hep.nd.edu

The narrative of these lectures contains three main threads: (i) CP violation despite having so far been observed only in the decays of neutral kaons has been recognized as a phenomenon of truly fundamental importance. The KM ansatz constitutes the minimal implementation of CP violation: without requiring unknown degrees of freedom it can reproduce the known CP phenomenology in a nontrivial way. (ii) The physics of beauty hadrons – in particular their weak decays – opens a novel window onto fundamental dynamics: they usher in a new quark family (presumably the last one); they allow us to determine fundamental quantities of the Standard Model like the b quark mass and the CKM parameters $V(cb)$, $V(ub)$, $V(ts)$ and $V(td)$; they exhibit speedy or even rapid $B^0 - \bar{B}^0$J oscillations. (iii) Heavy Quark Expansions allow us to treat B decays with an accuracy that would not have been thought possible a mere decade ago. These three threads are joined together in the following manner: (a) Huge CP asymmetries are *pre*dicted in B decays, which represents a decisive test of the KM paradigm for CP violation. (b) Some of these predictions are made with high *parametric* reliability, which (c) can be translated into *numerical* precision through the judicious employment of novel theoretical technologies. (d) Beauty decays thus provide us with a rich and promising field to search for New Physics and even study some of its salient features. At the end of it there might quite possibly be a New Paradigm for High Energy Physics. There will be some other threads woven into this tapistry: electric dipole moments, and CP violation in other strange and in charm decays.

Contents

[a] Invited Lectures given at the First APCTP Hadron Physics Winter School, Feb. 23 - 27, 1998, APCTP, Seoul, Korea

Schläft ein Lied in allen Dingen,
die da träumen fort und fort,
und die Welt hebt an zu singen,
findst Du nur das Zauberwort.

Sleeps a song in all things
that dream on and on
and the world will start to sing
if you find the magic word.

J. v. Eichendorff

1 Prologue

With very few symmetries in nature manifestly realized, why do I think that the breaking of CP invariance is very special – more subtle, more fundamental and more profound than parity violation?

- Parity violation tells us that nature makes a difference between "left" and "right" – but not which is which! For the statement that neutrinos emerging from pion decays are left- rather than right-handed implies the use of positive instead of negative pions. "Left" and "right" is thus defined in terms of "positive" and "negative", respectively. This is like saying that your left thumb is on your right hand – certainly correct, yet circular and thus not overly useful.

  On the other hand CP violation manifesting itself through

$$\frac{\mathrm{BR}(K_L \to l^+ \nu \pi^-)}{\mathrm{BR}(K_L \to l^- \bar{\nu} \pi^+)} \simeq 1.006J \neq 1 \tag{1}$$

  allows us to define "positive" and "negative" in terms of *observation* rather than *convention*, and subsequently likewise for "left" and "right".

- The limitation on CP invariance in the $K^0 - \bar{K}^0$ mass matrix

$$\mathrm{Im} M_{12} \simeq 1.1 \cdot 10^{-8} \ \mathrm{eV} \ \hat{=} \ \frac{\mathrm{Im} M_{12}}{m_K} \simeq 2.2 \cdot 10^{-17} \tag{2}$$

  represents the most subtle symmetry violation actually observed to date.

- CP violation constitutes one of the three essential ingredients in any attempt to understand the observed baryon number of the universe as a dynamically generated quantity rather than an initial condition [1].

- Due to CPT invariance – which will be assumed throughout these lectures – CP breaking implies a violation of time reversal invariance [b]. That nature makes an intrinsic distinction between past and future on the *microscopic* level that cannot be explained by statistical considerations is an utterly amazing observation.

- The fact that time reversal represents a very peculiar operation can be expressed also in a less emotional way, namely through *Kramers' Degeneracy* [2]. Since the time reversal operator $\mathbf{T}$ has to be *anti*-unitary, $\mathbf{T}^2$ has eigenvalues ± 1. Consider the sector of the Hilbert space with $\mathbf{T}^2 = -1$ and assume the dynamics to conserve $\mathbf{T}$; i.e., the Hamilton operator $\mathbf{H}$ and $\mathbf{T}$ commute. It is easily shown that if $|E\rangle$ is an eigenvector of $\mathbf{H}$, so is $\mathbf{T}|E\rangle$ – with the *same* eigenvalue. Yet $|E\rangle$ and $\mathbf{T}|E\rangle$ are – that is the main substance of this theorem – orthogonal to each other! Each energy eigenstate in the Hilbert sector with $\mathbf{T}^2 = -1$ is therefore at least doubly degenerate. This degeneracy is realized in nature through *fermionic spin* degrees. Yet it is quite remarkable that the time reversal operator $\mathbf{T}$ already anticipates this option – and the qualitative difference between fermions and bosons – through $\mathbf{T}^2 = \pm 1$ – *without* any explicit reference to spin!

[b] *Operationally* one defines time reversal as the reversal of *motion*: $\vec{p} \to -\vec{p}$, $\vec{j} \to -\vec{j}$ for momenta $\vec{p}$ and angular momenta $\vec{j}$.

These lectures will be organized as follows: in Lectures I and II – covered in Sect. 2 - 4 – I will list the existing CP phenomenology, introduce the KM ansatz as the minimal implementation of CP violation and apply it to $\Delta S = 2, 1$ transitions; in Lecture III – Sect. 5 - 6 – I describe in some detail CP violation in beauty and charm decays, both from the perspective of the KM ansatz as well as New Physics; in Lecture IV – Sect. 7 – I describe Heavy Quark Expansions and their applications to beauty decays before giving a summary and presenting an outlook.

While Lectures I - III represent a slight up-date of previous lectures [3], Lecture IV contains also a review of some very recent work on topical issues like quark-hadron duality.

2 CP Phenomenology in K_L Decays

2.1 Symmetries and Particle-Antiparticle Oscillations

A symmetry **S** can *manifestly* be realized in two different ways:

- There exists a pair of degenerate states that transform into each other under **S**.
- When there is an *un*paired state it has to be an eigenstate of **S**.

The observation of K_L decaying into a 2π state – which is CP even – and a CP odd 3π combination therefore establishes CP violation only because K_L and K_S are *not* mass degenerate.

In general, decay rates can exhibit CP violation in three different manners, namely through

1. a *difference* in CP conjugate rates, like $K_L \to l^- \bar{\nu} \pi^+$ vs. $K_L \to l^+ \nu \pi^-$,
2. the *existence* of a reaction, like $K_L \to \pi\pi$,
3. a decay rate evolution that is *not a purely exponential* function of the proper time of decay; i.e., if one finds for a CP *eigenstate* f

$$\frac{d}{dt} e^{\Gamma t} \text{rate}(K_{neutral}(t) \to f) \neq 0 \tag{3}$$

   for all (real) values of Γ, then CP symmetry must be broken. This is easily proven: if CP invariance holds, the decaying state must be a CP eigenstate like the final state f; yet in that case the decay rate evolution must be purely exponential – unless CP is violated. Q.E.D.

That the first manner represents CP violation is obvious without further ado. The situation is a bit more subtle with respect to the other two: the second relies on the mass eigenstate not being a CP eigenstate and third one on flavour eigenstates not being mass eigenstates. That means the latter two categories involve particle-antiparticle oscillations in an essential way.

The whole formalism of particle-antiparticle oscillations is actually a straightforward application of basic quantum mechanics. I will describe it in terms of

strange mesons; the generalization to any other flavour or quantum number (like beauty or charm) is straightforward and will be given later.

As long as CP is conserved, all relevant expressions can be given without having to tackle a differential equation explicitely. In the absence of weak forces one has two mass degenerate and stable mesons K^0 and $\bar{K}^0$ carrying definite strangeness $+1$ and -1, respectively, since the strong and electromagnetic forces conserve this quantum number. The addition of the weak forces changes the picture qualitatively: strangeness is no longer conserved, kaons become unstable and the new mass eigenstates – being linear superpositions of K^0 and $\bar{K}^0$ – no longer carry definite strangeness. The violation of the quantum number strangeness has lifted the degeneracy: we have two physical states K_L and K_S with different masses and lifetimes: $\Delta m_K = m_L - m_K \neq 0 \neq \Delta\tau = \tau_L - \tau_S$.

The mass eigenstates K_A and K_B have to be CP eigenstates as pointed out above: $|K_A\rangle = |K_+\rangle$, $|K_B\rangle = |K_-\rangle$, where $\mathbf{CP}|K_\pm\rangle \equiv \pm|K_\pm\rangle$. Using the phase convention

$$|\bar{K}^0\rangle \equiv -\mathbf{CP}|K^0\rangle \tag{4}$$

the time evolution of a state that starts out as a K^0 is given by

$$|K^0(t)\rangle = \frac{1}{\sqrt{2}} e^{-im_1 t} e^{-\frac{\Gamma_1}{2}t} \left(|K_+\rangle + e^{-i\Delta m_K t} e^{-\frac{\Delta\Gamma}{2}t}|K_-\rangle\right) \tag{5}$$

The intensity of an initially pure K^0 beam traveling in vacuum will then exhibit the following time profile:

$$I_{K^0}(t) = |\langle K^0|K^0(t)\rangle|^2 = \frac{1}{4} e^{-\Gamma_1 t} \left(1 + e^{\Delta\Gamma_K t} + 2e^{\frac{\Delta\Gamma_K}{2}t} \cos\Delta m_K t\right) \tag{6}$$

The orthogonal state $|\bar{K}^0(t)\rangle$ that was absent initially in this beam gets regenerated *spontaneously*:

$$I_{\bar{K}^0}(t) = |\langle \bar{K}^0|K^0(t)\rangle|^2 = \frac{1}{4} e^{-\Gamma_1 t} \left(1 + e^{\Delta\Gamma_K t} - 2e^{\frac{\Delta\Gamma_K}{2}t} \cos\Delta m_K t\right) \tag{7}$$

The oscillation rate expressed through Δm_K and $\Delta\Gamma_K$ is naturally calibrated by the average decay rate $\bar{\Gamma}_K \equiv \frac{1}{2}(\Gamma_1 + \Gamma_2)$:

$$x_K \equiv \frac{\Delta m_K}{\bar{\Gamma}_K} \simeq 0.95 \quad , \quad y_K \equiv \frac{\Delta\Gamma_K}{2\bar{\Gamma}_K} \simeq 1 \tag{8}$$

Two comments are in order at this point:

- In any such binary quantum system there will be two lifetimes. The fact that they differ so spectacularly for neutral kaons – $\tau(K_L) \sim 600 \cdot \tau(K_S)$ – is due to a kinematical accident: the only available nonleptonic channel for the CP odd kaon is the 3 pion channel, for which it has barely enough mass.

- $\Delta m_K \simeq 3.7 \cdot 10^{-6}$ eV is often related to the kaon mass:

$$\frac{\Delta m_K}{m_K} \simeq 7 \cdot 10^{-15} \tag{9}$$

which is obviously a very striking number. Yet Eq.(9) somewhat overstates the point. The kaon mass has nothing really to do with the $K_L - K_S$ mass difference [c]and actually is measured relative to Γ_K. There is however one exotic application where it makes sense to state the ratio $\Delta m_K/m_K$, and that is in the context of antigravity where one assumes matter and antimatter to couple to gravity with the opposite sign. The gravitational potential Φ would then produce a *relative* phase between K^0 and $\bar{K}^0$ of $2\ m_K\Phi t$. In the earth's potential this would lead to a gravitational oscillation time of 10^{-15} sec, which is much shorter than the lifetimes or the weak oscillation time; $K^0 - \bar{K}^0$ oscillations could then not be observed [4]. There are some loopholes in this argument – yet I consider it still intriguing or at least entertaining.

2.2 General Formalism

Oscillations become more complex once CP symmetry is broken in $\Delta S = 2$ transitions. The relevant formalism describes a general quantum mechanical situation. Consider a neutral meson P with flavour quantum number F; it can denote a K^0 or B^0. The time evolution of a state being a mixture of P and $\bar{P}$ is obtained from solving the (free) Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix} P^0 \\ \bar{P}^0 \end{pmatrix} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}\begin{pmatrix} P^0 \\ \bar{P}^0 \end{pmatrix} \tag{10}$$

CPT invariance imposes

$$M_{11} = M_{22} \ \ , \ \ \Gamma_{11} = \Gamma_{22} \ . \tag{11}$$

♠ ♠ ♠

Homework Problem #1:

Which physical situation is described by an equation analogous to Eq.(10) where however the two diagonal matrix elements differ *without* violating CPT?

♠ ♠ ♠

The subsequent discussion might strike the reader as overly technical, yet I hope she or he will bear with me since these remarks will lay important groundwork for a proper understanding of CP asymmetries in B decays as well.

The mass eigenstates obtained through diagonalising this matrix are given by (for details see [5,6])

$$|P_A\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}}\left(p|P^0\rangle + q|\bar{P}^0\rangle\right) \tag{12}$$

$$|P_B\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}}\left(p|P^0\rangle - q|\bar{P}^0\rangle\right) \tag{13}$$

with eigenvalues

$$M_A - \frac{i}{2}\Gamma_A = M_{11} - \frac{i}{2}\Gamma_{11} + \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right) \tag{14}$$

$$M_B - \frac{i}{2}\Gamma_B = M_{11} - \frac{i}{2}\Gamma_{11} - \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right) \tag{15}$$

[c]It would not be much more absurd to relate Δm_K to the mass of an elephant!

as long as

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \tag{16}$$

holds. I am using letter subscripts A and B for labeling the mass eigenstates rather than numbers 1 and 2 as it is usually done. For I want to avoid confusing them with the matrix indices $1, 2$ in $M_{ij} - \frac{i}{2}\Gamma_{ij}$ for reasons that will become clearer later.

Eqs.(15) yield for the differences in mass and width

$$\Delta M \equiv M_B - M_A = -2\mathrm{Re}\left[\frac{q}{p}(M_{12} - \frac{i}{2}\Gamma_{12})\right] \tag{17}$$

$$\Delta\Gamma \equiv \Gamma_A - \Gamma_B = -2\mathrm{Im}\left[\frac{q}{p}(M_{12} - \frac{i}{2}\Gamma_{12})\right] \tag{18}$$

Note that the subscripts A, B have been swapped in going from ΔM to $\Delta\Gamma$! This is done to have both quantities *positive* for kaons.

In expressing the mass eigenstates P_A and P_B explicitely in terms of the flavour eigenstates – Eqs.(13) – one needs $\frac{q}{p}$. There are two solutions to Eq.(16):

$$\frac{q}{p} = \pm\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \tag{19}$$

There is actually a more general ambiguity than this binary one. For antiparticles are defined up to a phase only:

$$\mathbf{CP}|P^0\rangle = \eta|\bar{P}^0\rangle \quad \text{with } |\eta| = 1 \tag{20}$$

Adopting a different phase convention will change the phase for $M_{12} - \frac{i}{2}\Gamma_{12}$ as well as for q/p:

$$|\bar{P}^0\rangle \to e^{i\xi}|\bar{P}^0\rangle \implies (M_{12}, \Gamma_{12}) \to e^{i\xi}(M_{12}, \Gamma_{12}) \;\&\; \frac{q}{p} \to e^{-i\xi}\frac{q}{p}\,, \tag{21}$$

yet leave $(q/p)(M_{12} - \frac{i}{2}\Gamma_{12})$ invariant – as it has to be since the eigenvalues, which are observables, depend on this combination, see Eq.(15). Also $\left|\frac{q}{p}\right|$ is an observable; its *deviation* from unity is one measure of CP violation in $\Delta F = 2$ dynamics.

By *convention* most authors pick the *positive* sign in Eq.(19)

$$\frac{q}{p} = +\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}\,. \tag{22}$$

Up to this point the two states $|P_{A,B}\rangle$ are merely *labelled* by their subscripts. Indeed $|P_A\rangle$ and $|P_B\rangle$ switch places when selecting the minus rather than the plus sign in Eq.(19).

One can define the labels A and B such that

$$\Delta M \equiv M_B - M_A > 0 \tag{23}$$

is satisfied. Once this *convention* has been adopted, it becomes a sensible question whether

$$\Gamma_B > \Gamma_A \quad \text{or} \quad \Gamma_B < \Gamma_A \tag{24}$$

holds, i.e. whether the heavier state is shorter or longer lived.

In the limit of CP invariance there is more we can say: since the mass eigenstates are CP eigenstates as well, we can raise another meaningful question: is the heavier state CP even or odd? With CP invariance requiring $\arg\frac{\Gamma_{12}}{M_{12}} = 0$ we have $\left|\frac{q}{p}\right| = 1$, i.e. $\frac{q}{p}$ becomes a pure phase. It is then convenient to adopt a phase convention s.t. M_{12} is real; it leads to

$$\frac{q}{p} = \pm 1 \tag{25}$$

Likewise we still have the freedom to choose between

$$\mathbf{CP}|P^0\rangle = +|\bar{P}^0\rangle \quad \text{or} \quad \mathbf{CP}|P^0\rangle = -|\bar{P}^0\rangle \tag{26}$$

Let us consider various choices:

- With $\frac{q}{p} = 1$ and $\mathbf{CP}|P^0\rangle = |\bar{P}^0\rangle$ we have

$$|P_A\rangle = \frac{1}{\sqrt{2}}\left(|P^0\rangle + |\bar{P}^0\rangle\right) = |P_+\rangle \tag{27}$$

$$|P_B\rangle = \frac{1}{\sqrt{2}}\left(|P^0\rangle - |\bar{P}^0\rangle\right) = |P_-\rangle \tag{28}$$

  with P_A and P_B being CP even and odd, respectively: $\mathbf{CP}|P_\pm\rangle = \pm|P_\pm\rangle$.

$$M_{odd} - M_{even} = M_B - M_A = -2\mathrm{Re}\left[\frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\right] = -2M_{12} \tag{29}$$

- Alternatively we can set $\frac{q}{p} = -1$

$$|P_A\rangle = \frac{1}{\sqrt{2}}\left(|P^0\rangle - |\bar{P}^0\rangle\right) = |P_-\rangle \tag{30}$$

$$|P_B\rangle = \frac{1}{\sqrt{2}}\left(|P^0\rangle + |\bar{P}^0\rangle\right) = |P_+\rangle \tag{31}$$

  while maintaining $\mathbf{CP}|P^0\rangle = +|\bar{P}^0\rangle$. P_A and P_B then switch roles; i.e., they are now CP odd and even, respectively. Accordingly:

$$M_{odd} - M_{even} = M_A - M_B = 2\mathrm{Re}\left[\frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\right] = -2M_{12} \tag{32}$$

- Finally let us consider choosing $\frac{q}{p} = 1$ together with $\mathbf{CP}|P^0\rangle = -|\bar{P}^0\rangle$. P_A and P_B are again expressed by Eq.(28), yet now are CP odd and even. Then

$$M_{odd} - M_{even} = M_B - M_A = 2M_{12} \tag{33}$$

- Eqs.(29,32) on one hand and Eq.(33) do not coincide on the surface. Yet we will see below that the theoretical expression for M_{12} changes sign depending on the choice of $\mathbf{CP}|P^0\rangle = \pm|\bar{P}^0\rangle$. Thus they all agree – as they have to!

- It is attractive to write the general mass eigenstates in terms of the CP eigenstates as well:

$$|P_A\rangle = \frac{1}{\sqrt{1+|\bar{\epsilon}|^2}}\left(|P_+\rangle + \bar{\epsilon}|P_-\rangle\rangle\right) \quad , \; CP|P_\pm\rangle = \pm|P_\pm\rangle \tag{34}$$

$$|P_B\rangle = \frac{1}{\sqrt{1+|\bar{\epsilon}|^2}}\left(|P_-\rangle + \bar{\epsilon}|P_+\rangle\rangle\right) \; ; \tag{35}$$

  $\bar{\epsilon} = 0$ means that the mass and CP eigenstates coincide, i.e. CP is conserved in $\Delta F = 2$ dynamics driving $P - \bar{P}$ oscillations. With the phase between the orthogonal states $|P_+\rangle$ and $|P_-\rangle$ arbitrary, the phase of $\bar{\epsilon}$ can be changed at will and is not an observable; $\bar{\epsilon}$ can be expressed in terms of $\frac{q}{p}$, yet in a way that depends on the convention for the phase of antiparticles. For $\mathbf{CP}|P\rangle = |\bar{P}\rangle$ one has

$$|P_+\rangle = \frac{1}{\sqrt{2}}\left(|P^0\rangle + |\bar{P}^0\rangle\right) \tag{36}$$

$$|P_-\rangle = \frac{1}{\sqrt{2}}\left(|P^0\rangle - |\bar{P}^0\rangle\right) \tag{37}$$

$$\bar{\epsilon} = \frac{1-\frac{q}{p}}{1+\frac{q}{p}} \tag{38}$$

  whereas for $\mathbf{CP}|P\rangle = -|\bar{P}\rangle$ one finds

$$|P_+\rangle = \frac{1}{\sqrt{2}}\left(|P^0\rangle - |\bar{P}^0\rangle\right) \tag{39}$$

$$|P_-\rangle = \frac{1}{\sqrt{2}}\left(|P^0\rangle + |\bar{P}^0\rangle\right) \tag{40}$$

$$\bar{\epsilon} = \frac{1+\frac{q}{p}}{1-\frac{q}{p}} \tag{41}$$

- The lack of orthogonality between P_A and P_B is a measure of CP violation in $\Delta F = 2$ dynamics:

$$\langle P_B|P_A\rangle = \frac{1-\left|\frac{q}{p}\right|^2}{1+\left|\frac{q}{p}\right|^2} = \frac{2\mathrm{Re}\bar{\epsilon}}{1+|\bar{\epsilon}|^2} \tag{42}$$

Later we will discuss how to evaluate M_{12} and thus also ΔM within a given theory for the $P - \bar{P}$ complex. The examples just listed illustrate that some care has to be applied in interpreting such results. For expressing mass eigenstates explicitely

in terms of flavour eigenstates involves some conventions. Once adopted we have to stick with a convention; yet our original choice cannot influence observables.

We had already referred to the fact that the *relative* phase between Γ_{12} and M_{12} represents an observable describing indirect CP violation. Therefore we adopt the notation

$$M_{12} = \bar{M}_{12} e^{i\xi} \ , \ \ \Gamma_{12} = \bar{\Gamma}_{12} e^{i\xi} e^{i\zeta} \ \text{ and } \frac{\Gamma_{12}}{M_{12}} = \frac{\bar{\Gamma}_{12}}{\bar{M}_{12}} e^{i\zeta} \equiv r e^{i\zeta} \tag{43}$$

We restrict the angles ξ and $\xi + \zeta$ to lie between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$; i.r., the real quantities $\bar{M}_{12}$ and $\bar{\Gamma}_{12}$ are a priori allowed to be *negative* as well as *positive*! A relative minus sign between M_{12} and Γ_{12} is of course physically significant, while the absolute sign is not. Yet it turns out that the absolute sign provides us with a useful though dispensible bookkeeping device.

Let me recapitulate the relevant points:

- The labels of the two mass eigenstates P_A and P_B can be chosen such that
$$M_{P_B} > M_{P_A} \tag{44}$$
holds.
- Then it becomes an *empirical* question whether P_A or P_B are longer lived:
$$\Gamma_{P_A} > \Gamma_{P_B} \quad \text{or} \quad \Gamma_{P_A} < \Gamma_{P_B} \ \ ? \tag{45}$$
- In the limit of CP invariance one can also raise the question whether it is the CP even or the odd state that is heavier.
- We will see later that within a *given theory* for $\Delta F = 2$ dynamics one can calculate M_{12}, including its sign, if phase conventions are treated consistently. To be more specific: adopting a phase convention for $\frac{q}{p}$ and having $\mathcal{L}(\Delta F = 2)$ one can calculate $\frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right) = \frac{q}{p}\langle P^0|\mathcal{L}(\Delta F = 2)|\bar{P}^0\rangle$. Then one assigns the labels B and A such that $\Delta M = M_B - M_A = -2\text{Re}\frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)$ turns out to be *positive*!

2.3 The $K^0 - \bar{K}^0$ Complex

For the kaon system I have already stated the observed values for Δm_K and $\Delta\Gamma_K$ in Eq.(8); using the convention of Eq.(23) – $\Delta m_K = M_B - M_A > 0$ – the data tell us

$$\Delta\Gamma_K = \Gamma_A - \Gamma_B > 0 \ ; \tag{46}$$

i.e., the (ever so slightly) heavier neutral kaon is (considerably) longer lived [d] and it is approximately the CP odd state. With CP violation small – $\zeta_K = \arg\left(\Gamma_{12}^K/M_{12}^K\right) \ll 1$ – one deduces from Eqs.(18) in the notation of Eq.(43) with the convention of Eq.(22):

$$\Delta m_K \simeq -2\bar{M}_{12}^K \ , \ \Delta\Gamma_K \simeq 2\bar{\Gamma}_{12}^K \tag{47}$$

Let me add a few comments that apply specifically here:

[d] The English language provides us with the convenient mnemonic that the subscript L denotes both *l*onger in lifetime and *l*arger in mass.

- On very general grounds – without recourse to any model – one can infer that CP violation in the neutral kaon system has to be small. The *Bell-Steinberger relation* allows to place a bound on the scalar product of the two mass eigenstates, introduced in Eq.(42) [5,6]:

$$\langle K_L|K_S\rangle \leq \sqrt{2}\sum_f \sqrt{\frac{\Gamma_L^f \Gamma_S^f}{\Gamma_S^2}} \leq \sqrt{2}\sqrt{\frac{\Gamma_L}{\Gamma_S}} \simeq 0.06 \tag{48}$$

  There is no input from any CP measurement. What is essential, though, is the huge lifetime ratio.

- There are actually two processes underlying the transition $K_L \to 2\pi$: $\Delta S = 2$ forces generate the mass eigenstates K_L and K_S whereas $\Delta S = 1$ dynamics drive the decays $K \to 2\pi$. Thus CP violation can enter in two a priori independant ways, namely through the $\Delta S = 2$ and the $\Delta S = 1$ sector. This distinction can be made explicit in terms of the transition amplitudes:

$$\eta_{+-} \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \equiv \epsilon_K + \epsilon' \ , \ \eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \equiv \epsilon_K - 2\epsilon' \tag{49}$$

  The quantity ϵ_K describes the CP violation common to the K_L decays; it thus characterizes the decaying *state* and is referred to as *CP violation in the mass matrix* or *superweak CP violation*; ϵ' on the other hand differentiates between different channels and thus characterizes *decay* dynamics; it is called *direct CP violation.*

- *Maximal* parity and/or charge conjugation violation can be defined by saying there is no right-handed neutrino and/or left-handed antineutrino, respectively. Yet *maximal* CP violation *cannot* be defined in an analogous way: for the existence of the right-handed antineutrino which is the CP conjugate to the left-handed neutrino is already required by CPT invariance.

2.4 Data

The data on CP violation in neutral kaon decays are as follows:

1. *Existence* of $K_L \to \pi\pi$:

$$\begin{array}{c} \mathrm{BR}(K_L \to \pi^+\pi^-) = (2.067 \pm 0.035) \cdot 10^{-3} \\ \mathrm{BR}(K_L \to \pi^0\pi^0) = (0.936 \pm 0.020) \cdot 10^{-3} \end{array} \tag{50}$$

2. Search for *direct* CP violation:

$$\frac{\epsilon'}{\epsilon_K} \simeq \mathrm{Re}\frac{\epsilon'}{\epsilon_K} = \begin{cases} (2.3 \pm 0.65) \cdot 10^{-3} & \mathit{NA\ 31} \\ (1.5 \pm 0.8) \cdot 10^{-3} & \mathit{PDG\ '96\ average} \\ (0.74 \pm 0.52 \pm 0.29) \cdot 10^{-3} & \mathit{E\ 731} \end{cases} \tag{51}$$

3. Rate *difference* in semileptonic decays:

$$\delta_l \equiv \frac{\Gamma(K_L \to l^+\nu\pi^-) - \Gamma(K_L \to l^-\bar{\nu}\pi^+)}{\Gamma(K_L \to l^+\nu\pi^-) + \Gamma(K_L \to l^-\bar{\nu}\pi^+)} = (3.27 \pm 0.12) \cdot 10^{-3} \; , \qquad (52)$$

where an average over electrons and muons has been taken.

4. *T violation*:

$$\frac{\Gamma(K^0 \Rightarrow \bar{K}^0) - \Gamma(\bar{K}^0 \Rightarrow K^0)}{\Gamma(K^0 \Rightarrow \bar{K}^0) + \Gamma(\bar{K}^0 \Rightarrow K^0)} = (6.3 \pm 2.1 \pm 1.8) \cdot 10^{-3} \quad CPLEAR \qquad (53)$$

from a third of their data set [7]. It would be premature to claim this asymmetry has been established; yet it represents an intriguingly direct test of time reversal violation and is sometimes referred to as the Kabir test . It requires tracking the flavour identity of the *decaying* meson as a K^0 or $\bar{K}^0$ through its semileptonic decays – $\bar{K}^0 \to l^-\bar{\nu}\pi^+$ vs. $K^0 \to l^+\nu\pi^-$ – and also of the *initially produced* kaon. The latter is achieved through correlations imposed by associated production. The CPLEAR collaboration studied low energy proton-antiproton annihilation

$$p\bar{p} \to K^+\bar{K}^0\pi^- \;\; vs. \;\; p\bar{p} \to K^-K^0\pi^+ \; ; \qquad (54)$$

the charged kaon reveals whether a K^0 or a $\bar{K}^0$ was produced in association with it. In the future the CLOE collaboration will study T violation in $K^0\bar{K}^0$ production at DAΦNE:

$$e^+e^- \to \phi(1020) \to K^0\bar{K}^0 \qquad (55)$$

5. CP- and T-odd Correlations:

The KTeV Collaboration at Fermilab has established the existence of a new rare K_L decay mode [e]:

$$BR(K_L \to \pi^+\pi^-e^+e^-) = (3.32 \pm 0.14 \pm 0.28) \cdot 10^{-7} \qquad (56)$$

With ϕ defined as the angle between the planes spanned by the two pions and the two leptons in the K_L restframe:

$$\phi \equiv \angle(\vec{n}_l, \vec{n}_\pi)$$

$$\vec{n}_l = \vec{p}_{e^+} \times \vec{p}_{e^-}/|\vec{p}_{e^+} \times \vec{p}_{e^-}| \; , \; \vec{n}_\pi = \vec{p}_{\pi^+} \times \vec{p}_{\pi^-}/|\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}| \qquad (57)$$

one analyzes the decay rate as a function of ϕ:

$$\frac{d\Gamma}{d\phi} = \Gamma_1 \cos^2\phi + \Gamma_2 \sin^2\phi + \Gamma_3 \cos\phi \sin\phi \qquad (58)$$

Since

$$\cos\phi \sin\phi = (\vec{n}_l \times \vec{n}_\pi) \cdot (\vec{p}_{\pi^+} + \vec{p}_{\pi^-})(\vec{n}_l \cdot \vec{n}_\pi)/|\vec{p}_{\pi^+} + \vec{p}_{\pi^-}| \qquad (59)$$

[e] Obviously these data were not available at the actual lectures.

one notes that

$$\cos\phi\sin\phi \quad \stackrel{\mathbf{T,CP}}{\longrightarrow} \quad -\cos\phi\sin\phi \tag{60}$$

under both T and CP transformations; i.e. the observable Γ_3 represents a T- and CP-odd correlation. It can be projected out by comparing the ϕ distribution integrated over two quadrants:

$$A = \frac{\int_0^{\pi/2} d\phi \frac{d\Gamma}{d\phi} - \int_{\pi/2}^{\pi} d\phi \frac{d\Gamma}{d\phi}}{\int_0^{\pi} d\phi \frac{d\Gamma}{d\phi}} = \frac{2\Gamma_3}{\pi(\Gamma_1 + \Gamma_2)} \tag{61}$$

KTeV observes

$$A = (13.5 \pm 2.5 \pm 3.0)\% \, , \text{ preliminary} \tag{62}$$

This represents a new world record for the size of a CP asymmetry.

2.5 Phenomenological Interpretation

Semileptonic Transitions

CPT symmetry imposes constraints well beyond the equality of lifetimes for particles and antiparticles: certain *sub*classes of decay rates have to be equal as well. For example one finds

$$\Gamma(\bar{K}^0 \to l^- \bar{\nu} \pi^+) = \Gamma(K^0 \to l^+ \nu \pi^-) \tag{63}$$

The rate asymmetry in semileptonic decays listed in Eq.(52) thus reflects pure superweak CP violation:

$$\delta_l = \frac{1 - |q/p|^2}{1 + |q/p|^2} \tag{64}$$

From the measured value of δ_l one then obtains

$$\left| \frac{q}{p} \right| = 1 + (3.27 \pm 0.12) \cdot 10^{-3} \tag{65}$$

Since one has for the $K^0 - \bar{K}^0$ system specifically

$$\left| \frac{q}{p} \right| \simeq 1 + \frac{1}{2} \arg \frac{M_{12}}{\Gamma_{12}} \tag{66}$$

one can express this kind of CP violation through a phase:

$$\Phi(\Delta S = 2) \equiv \arg \frac{M_{12}}{\Gamma_{12}} = (6.54 \pm 0.24) \cdot 10^{-3} \tag{67}$$

The result of the Kabir test, Eq.(53), yields:

$$\Phi(\Delta S = 2) = (6.3 \pm 2.1 \pm 1.8) \cdot 10^{-3} \, , \tag{68}$$

which is of course consistent with Eq.(67).

Using the measured value of $\Delta m_K / \Delta\Gamma_K$ one infers

$$\frac{M_{12}}{\Gamma_{12}} = -(0.4773 \pm 0.0023) \left[1 - i(6.54 \pm 0.24) \cdot 10^{-3} \right] \tag{69}$$

Nonleptonic Transitions

From Eq.(50) one deduces

$$\begin{aligned} |\eta_{+-}| &= (2.275 \pm 0.019) \cdot 10^{-3} \\ |\eta_{00}| &= (2.285 \pm 0.019) \cdot 10^{-3} \end{aligned} \tag{70}$$

As mentioned before the ratios $\eta_{+-,00}$ are sensitive also to direct CP violation generated by a phase between the decay amplitudes $A_{0,2}$ for $K_L \to (\pi\pi)_I$, where the subscript I denotes the isospin of the 2π system:

$$\Phi(\Delta S = 1) \equiv \arg \frac{A_2}{A_0} \tag{71}$$

One finds

$$\eta_{+-} \simeq \frac{i\tilde{x}}{2\tilde{x}+i} \left[\Phi(\Delta S = 2) + 2\omega \Phi(\Delta S = 1) \right] \ , \tag{72}$$

with

$$\tilde{x} \equiv \frac{\Delta m_K}{\Delta \Gamma_K} = \frac{\Delta m_K}{\Gamma(K_S)} = \frac{1}{2} x_K \simeq 0.477 \ \ , \ \ \omega \equiv \left| \frac{A_2}{A_0} \right| \simeq 0.05 \tag{73}$$

where the second quantity represents the observed enhancement of A_0 for which a name – "$\Delta I = 1/2$ rule" – yet no quantitative dynamical explanation has been found. Equivalently one can write

$$\frac{\epsilon'}{\epsilon_K} \simeq 2\omega \frac{\Phi(\Delta S = 1)}{\Phi(\Delta S = 2)} \tag{74}$$

The data on $K_L \to \pi\pi$ can thus be expressed as follows [8]

$$\begin{aligned} \Phi(\Delta S = 2) &= (6.58 \pm 0.26) \cdot 10^{-3} \\ \Phi(\Delta S = 1) &= (0.99 \pm 0.53) \cdot 10^{-3} \end{aligned} \tag{75}$$

Radiative Transitions

The modes $K_{L,S} \to \pi^+\pi^-\gamma$ have been observed with

$$BR(K_L \to \pi^+\pi^-\gamma) = (4.66 \pm 0.15) \cdot 10^{-5} \tag{76}$$

$$BR(K_S \to \pi^+\pi^-\gamma) = (4.87 \pm 0.11) \cdot 10^{-3} \tag{77}$$

for $E_\gamma > 20$ MeV. Two mechanisms can drive these channels and an analysis of the photon spectra indeed reveals the intervention of both:

- Bremsstrahlung off the pions through an E1 transition:

  $$K_L \stackrel{\Delta S=1}{\longrightarrow} \pi^+\pi^- \stackrel{E1}{\longrightarrow} \pi^+\pi^-\gamma \ , \ K_S \stackrel{\Delta S=1}{\longrightarrow} \pi^+\pi^- \stackrel{E1}{\longrightarrow} \pi^+\pi^-\gamma \tag{78}$$

  where only the first step in the K_L decay is CP violating.

- Direct photon emission of the M1 type

  $$K_L \stackrel{M1\&\Delta S=1}{\longrightarrow} \pi^+\pi^-\gamma \ , \ K_S \stackrel{M1\&\Delta S=1}{\longrightarrow} \pi^+\pi^-\gamma \ , \tag{79}$$

  which is CP conserving [violating] for the $K_L[K_S]$ process.

In analogy to η_{+-} one defines a ratio of E1 amplitudes

$$\eta_{+-\gamma} = \frac{T(K_L \to \pi^+\pi^-\gamma, E1)}{T(K_S \to \pi^+\pi^-\gamma, E1)} \tag{80}$$

that measures CP violation. Without *direct* CP violation one has $\eta_{+-\gamma} = \eta_{+-}$.

The interference of the CP violating E1 and conserving M1 amplitudes for $K_L \to \pi^+\pi^-\gamma$ will yield a circularly polarized photon. To be more explicit: it yields a triple correlation between the pion momenta and the photon polarization

$$P_\perp^\gamma = \langle \vec{\epsilon}_\gamma \cdot (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}) \rangle \tag{81}$$

which is CP-odd; its leading contribution is proportional to η_{+-} entering in the E1 amplitude.

This polarization can be probed best for off-shell photons

$$K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^- e^+ e^- \tag{82}$$

by measuring the correlation between the e^+e^- and $\pi^+\pi^-$ planes measured through the angle ϕ, see Eq.(57).

This transition has been analyzed several years ago in [9]. The transition amplitude reads as follows:

$$T(K_L \to \pi^+\pi^- e^+ e^-) = e|T(K_S \to \pi^+\pi^-)| \cdot$$
$$\cdot \left[g_{E1} \left(\frac{p_+^\mu}{p_+ \cdot k} - \frac{p_-^\mu}{p_- \cdot k} \right) + g_{M1} \epsilon_{\mu\nu\alpha\beta} k^\nu p_+^\alpha p_-^\beta \right] \frac{e}{k^2} \bar{u}(k_-)\gamma_\mu v(k_+) \tag{83}$$

with $k = k_+ + k_-$; the two couplings $g_{E1,M1}$ are given by

$$g_{E1} = \eta_{+-} e^{i\delta_0(m_K^2)} \;\; , \;\; g_{M1} = 0.76 i e^{i\delta_1(s_\pi)} \tag{84}$$

with $\delta_{0,1}$ denoting the s- and p-wave $\pi\pi$ phase shifts; the coefficient 0.76 is obtained from the observed branching ratio for the M1 transition. These expressions lead to the following predictions [f]

$$BR(K_L \to \pi^+\pi^- e^+ e^-) \simeq 3 \cdot 10^{-7} \tag{85}$$

depending on the cut one places on the e^+e^- invariant mass and

$$A \simeq (14.3 \pm 1.3)\% \tag{86}$$

The main theoretical uncertainty resides in what one assumes for the hadronic form factors. In [9] a phenomenological ansatz was employed; evaluating them in chiral perturbation theory yields similar numbers [10].

These predictions are in full agreement with the KTeV data, see Eqs.(56,62).

The discovery of such a large CP asymmetry is a significant result be it only to show that CP violation is not uniformly tiny in K_L decays. One should note, though, that A is driven by η_{+-} entering through $K_L \to \pi^+\pi^- \to \pi^+\pi^-\gamma^* \to \pi^+\pi^- e^+ e^-$; its size thus is not the prediction of a specific model. Two more comments are in order here:

---

[f] These are predictions in the old-fashioned way: they were stated before there were data.

- *Direct* CP violation can affect A as well; its size depends on the specifics of the dynamics underlying CP violation. Yet its contributions to A averaged over all final states are tiny, namely $< 10^{-3}$ for the KM ansatz [11]; it is hard to see how they could be significantly larger for other models. While such contributions could be considerably larger in certain parts of phase space, no promising avenue has been pointed out yet.
- The correlation A is clearly T-odd. Yet with the time reversal operator being *anti*unitary a T-odd correlation can arise even with T-invariant dynamics, if complex phases are present [6]; final state interactions can generate such phases. One might be tempted to argue that an observation of $A > 0.05$ *directly* establishes T violation: for *electromagnetic* final state interactions cannot generate an effect of that size while *strong* final state interactions can affect neither the photon polarization nor the orientation of the $\pi - \pi$ plane. There is a third possibility, though, in this special situation: CP violation induces an E1 amplitude proportional to η_{+-}; it contains an observable phase ϕ_{+-} that depends on the underlying physics. CPT invariance constrains ϕ_{+-} to be close to 45^o. In that scenario $\eta_{+-} \neq 0$ of course implies T as well as CP violation. As a matter of principle at least one can then ask what happens if both CP and CPT invariance are broken while T remains conserved; the phase of η_{+-} is then no longer constrained. One can then fit this phase and – as a point of principle – will find a solution for reproducing A with T invariant dynamics. In practise one can then check whether the value of ϕ_{+-} thus obtained is consistent with the findings from $K_L \to \pi\pi$ and $K_L \to l^{\pm}\nu\pi^{\mp}$. One thus interpretes A as a novel probe of CPT invariance

Resume

The experimental results can be summarized as follows:

- The decays of neutral kaons exhibit unequivocally CP violation of the superweak variety, which is expressed through the angle $\Phi(\Delta S = 2)$. The findings from semileptonic, nonleptonic and now even radiative transitions concur to an impressive and reassuring degree;
- Direct CP violation still has not been established.
- A theorist might be forgiven for mentioning that the evolution of the measurements over the last twenty odd years has not followed the straight line this brief summary might suggest to the uninitiated reader.

Theoretical Implementation of CP Violation

2.6 Some Historical Remarks

Theorists can be forgiven if they felt quite pleased with the state of their craft in 1964:

- The concept of (quark) families had emerged, at least in a rudimentary form.

- Maximal parity and charge conjugation violations had been found in weak charged current interactions, yet CP invariance apparently held. Theoretical pronouncements were made ex cathedra why this had to be so!

- *Pre*dictions of the existence of two kinds of neutral kaons with different lifetimes and masses had been confirmed by experiment [12].

That same year the reaction $K_L \to \pi^+\pi^-$ was discovered [13]! Two things should be noted here. The Fitch-Cronin experiment had predecessors: rather than being an isolated effort it was the culmination of a whole research program. Secondly there was at least one theoretical voice, namely that of Okun [14], who in 1962/63 had listed a dedicated search for $K_L \to \pi\pi$ as one of the most important unfinished tasks. Nevertheless for the vast majority of the community the Fitch-Cronin observation came as a shock and caused considerable consternation among theorists. Yet – to their credit – these data and their consequence, namely that CP invariance was broken, were soon accepted as facts. This was phrased – though *not explained* – in terms of the Superweak Model [15] later that same year.

In 1970 the renormalizability of the $SU(2)_L \times U(1)$ electroweak gauge theory was proven. I find it quite amazing that it was still not realized that the physics known at that time could not produce CP violation. As long as one had to struggle with infinities in the theoretical description one could be forgiven for not worrying unduly about a tiny quantity like $\mathrm{BR}(K_L \to \pi^+\pi^-) \simeq 2.3 \cdot 10^{-3}$. Yet no such excuse existed any longer once a renormalizable theory had been developed! The existence of the Superweak Model somewhat muddled the situation in this respect: for it provides merely a classification of the dynamics underlying CP violation rather than a dynamical description itself.

The paper by Kobayashi and Maskawa [16], written in 1972 and published in 1973, was the first

- to state clearly that the $SU(2)_L \times U(1)$ gauge theory even with two complete families [g] is necessarily CP-invariant and

- to list the possible extensions that could generate CP violation; among them – as one option – was the three (or more) family scenario now commonly referred to as the KM ansatz. They also discussed the impact of right-handed currents and of a non-minimal Higgs sector.

2.7 The Minimal Model: The KM Ansatz

Once a theory reaches a certain degree of complexity, many potential sources of CP violation emerge. Popular examples of such a scenario are provided by models implementing supersymmetry or its local version, supergravity; hereafter both are referred to as SUSY. In my lectures I will however focus on the minimal theory that can support CP violation, namely the Standard Model with three families. All of its dynamical elements have been observed – except for the Higgs boson, of course.

[g] Remember this was still before the J/ψ discovery!

Weak Phases like the Scarlet Pimpernel

Weak interactions at low energies are described by four-fermion interactions. The most general expression for spin-one couplings are

$$\mathcal{L}_{V/A} = \left(\bar{\psi}_1\gamma_\mu(a+b\gamma_5)\psi_2\right)\left(\bar{\psi}_3\gamma_\mu(c+d\gamma_5)\psi_4\right) + \\ + \left(\bar{\psi}_2\gamma_\mu(a^*+b^*\gamma_5)\psi_1\right)\left(\bar{\psi}_4\gamma_\mu(c^*+d^*\gamma_5)\psi_3\right) \tag{87}$$

Under CP these terms transform as follows:

$$\mathcal{L}_{V/A} \stackrel{CP}{\Longrightarrow} CP\mathcal{L}_{V/A}(CP)^\dagger = \left(\bar{\psi}_2\gamma_\mu(a+b\gamma_5)\psi_1\right)\left(\bar{\psi}_4\gamma_\mu(c+d\gamma_5)\psi_3\right) + \\ + \left(\bar{\psi}_1\gamma_\mu(a^*+b^*\gamma_5)\psi_2\right)\left(\bar{\psi}_3\gamma_\mu(c^*+d^*\gamma_5)\psi_4\right) \tag{88}$$

If a, b, c, d are real numbers, one obviously has $\mathcal{L}_{V/A} = CP\mathcal{L}_{V/A}(CP)^\dagger$ and CP is conserved. Yet CP is *not necessarily* broken if these parameters are complex, as we will explain specifically for the Standard Model.

Weak Universality arises naturally whenever the weak charged current interactions are described through a *single* non-abelian gauge group – $SU(2)_L$ in the case under study. For the single *self*-coupling of the gauge bosons determines also their couplings to the fermions; one finds for the quark couplings to the charged W bosons:

$$\mathcal{L}_{CC} = g\bar{U}_L^{(0)}\gamma_\mu D_L^{(0)}W^\mu + \bar{U}_R^{(0)}\mathbf{M}_U U_L^{(0)} + \bar{D}_R^{(0)}\mathbf{M}_D D_L^{(0)} + h.c. \tag{89}$$

where U and D denote the up- and down-type quarks, respectively:

$$U = (u, c, t) \quad , \quad D = (d, s, b) \tag{90}$$

and $\mathbf{M_U}$ and $\mathbf{M_D}$ their 3×3 mass matrices. In general those will not be diagonal; to find the physical states, one has to diagonalize these matrices:

$$\mathbf{M}_U^{diag} = \mathbf{K}_R^U\mathbf{M}_U(\mathbf{K}_L^U)^\dagger \ \ , \ \ \mathbf{M}_D^{diag} = \mathbf{K}_R^D\mathbf{M}_D(\mathbf{K}_L^D)^\dagger \tag{91}$$

$$U_{L,R} = \mathbf{K}_{L,R}^U U_{L,R}^{(0)} \ \ , \ \ D_{L,R} = \mathbf{K}_{L,R}^D D_{L,R}^{(0)} \tag{92}$$

with $\mathbf{K}_{L,R}^{U,D}$ representing four unitary 3×3 matrices. The coupling of these physical fermions to W bosons is then given by

$$\mathcal{L}_{CC} = g\bar{U}_L(\mathbf{K}_L^U)^\dagger\mathbf{K}_L^D\gamma_\mu DW^\mu + \bar{U}_R\mathbf{M}_U^{diag}U_L + \bar{D}_R\mathbf{M}_D^{diag}D_L + h.c. \tag{93}$$

and the combination $(\mathbf{K}_L^U)^\dagger\mathbf{K}_L^D \equiv \mathbf{V}_{CKM}$ represents the KM matrix, which obviously has to be unitary like K^U and K^D. Unless the up- and down-type mass matrices are aligned in flavour space (in which case they would be diagonalized by the same operators $\mathbf{K}_{L,R}$) one has $\mathbf{V}_{CKM} \neq 1$.

In the neutral current sector one has

$$\mathcal{L}_{NC} = g'\bar{U}_L^{(0)}\gamma_\mu U_L^{(0)}Z_\mu = g'\bar{U}_L\gamma_\mu U_L Z_\mu \tag{94}$$

and likewise for U_R and $D_{L,R}$; i.e. *no* flavour changing neutral currents are generated, let alone new phases. CP violation thus has to be embedded into the charged current sector.

If $\mathbf{V}_{CKM}$ is real (and thus orthogonal), CP symmetry is conserved in the weak interactions. Yet the occurrance of complex matrix elements does not *automatically* signal CP violation. This can be seen through a straightforward (in hindsight at least) algebraic argument. A unitary $N \times N$ matrix contains N^2 independant real parameters; $2N-1$ of those can be eliminated through re-phasing of the N up-type and N down-type fermion fields (changing all fermions by the *same* phase obviously does not affect $\mathbf{V}_{CKM}$). Hence there are $(N-1)^2$ real physical parameters in such an $N \times N$ matrix. For $N = 2$, i.e. two families, one recovers a familiar result, namely there is just one mixing angle, the Cabibbo angle. For $N = 3$ there are four real physical parameters, namely three (Euler) angles – and one phase. It is the latter that provides a gateway for CP violation. For $N = 4$ Pandora's box opens up: there would be 6 angles and 3 phases.

PDG suggests a "canonical" parametrization for the 3×3 CKM matrix:

$$\mathbf{V}_{CKM} = \begin{pmatrix} V(ud) & V(us) & V(ub) \\ V(cd) & V(cs) & V(cb) \\ V(td) & V(ts) & V(tb) \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{13}c_{23} \end{pmatrix} \quad (95)$$

where

$$c_{ij} \equiv \cos\theta_{ij} \quad , \quad s_{ij} \equiv \sin\theta_{ij} \quad (96)$$

with $i, j = 1, 2, 3$ being generation labels.

This is a completely general, yet not unique parametrisation: a different set of Euler angles could be chosen; the phases can be shifted around among the matrix elements by using a different phase convention. In that sense one can refer to the KM phase as the Scarlet Pimpernel: "Sometimes here, sometimes there, sometimes everywhere!"

Using just the observed hierarchy

$$|V(ub)| \ll |V(cb)| \ll |V(us)|, |V(cd)| \ll 1 \quad (97)$$

one can, as first realized by Wolfenstein, expand $\mathbf{V}_{CKM}$ in powers of the Cabibbo angle θ_C:

$$\mathbf{V}_{CKM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta + \frac{i}{2}\eta\lambda^2) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 - i\eta A^2\lambda^4 & A\lambda^2(1 + i\eta\lambda^2) \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^6) \quad (98)$$

where

$$\lambda \equiv \sin\theta_C \quad (99)$$

For such an expansion in powers of λ to be self-consistent, one has to require that $|A|$, $|\rho|$ and $|\eta|$ are of order unity. Numerically we obtain

$$\lambda = 0.221 \pm 0.002 \tag{100}$$

from $|V(us)|$,

$$A = 0.81 \pm 0.06 \tag{101}$$

from $|V(cb)| \simeq 0.040 \pm 0.002|_{exp} \pm 0.002|_{theor}$ and

$$\sqrt{\rho^2 + \eta^2} \sim 0.38 \pm 0.11 \tag{102}$$

from $|V(ub)| \sim (3.2 \pm 0.8) \cdot 10^{-3}$. The numbers for $|V(cb)|$ and $|V(ub)|$ have changed quite considerably over the last few years; in particular there has been a substantial reduction in the uncertainties. This reflects – in addition to more sensitive data of course – the arrival of novel theoretical technologies for dealing with heavy flavour decays. These methods will be briefly described in Sect.7.

We see that the CKM matrix is a very special unitary matrix: it is almost diagonal, it is almost symmetric and the matrix elements get smaller the more one moves away from the diagonal. Nature most certainly has encoded a profound message in this peculiar pattern. Alas – we have not succeeded yet in deciphering it! I will return to this point at the end of my lectures.

Unitarity Triangles

The qualitative difference between a two and a three family scenario can be seen also in a less abstract way. Consider $\bar{K}^0 \to \pi^+\pi^-$; it can proceed through a tree-level process $[s\bar{d}] \to [d\bar{u}][u\bar{d}]$, in which case its weak couplings are given by $V(us)V^*(ud)$. Or it can oscillate first to K^0 before decaying; i.e., on the quark level it is the transition $[s\bar{d}] \to [d\bar{s}] \to [d\bar{u}][u\bar{d}]$ controlled by $(V(cs))^2\,(V^*(cd))^2\,V^*(us)V(ud)$. At first sight it would seem that those two combinations of weak parameters are not only different, but should also exhibit a relative phase. Yet the latter is not so – if there are two families only! In that case the four quantities $V(ud)$, $V(us)$, $V(cd)$ and $V(cs)$ have to form a unitary 2×2 which leads to the constraint

$$V(ud)V^*(us) + V(cd)V^*(cs) = 0 \tag{103}$$

Using Eq.(103) twice one gets

$$\begin{aligned}(V(cs)V^*(cd))^2\,V^*(us)V(ud) &= -|V(cd)V(cs)|^2 V(cs)V^*(cd) = \\ &= |V(cd)V(cs)|^2 V^*(ud)V(us)\,;\end{aligned} \tag{104}$$

i.e., the two combinations $V^*(ud)V(us)$ and $(V(cs))^2\,(V^*(cd))^2\,V^*(us)V(ud)$ are actually parallel to each other with *no* relative phase. A penguin operator with a charm quark as the internal fermion line generates another contribution to $K_L \to \pi^-\pi^+$, this one controlled by $V(cs)V^*(cd)$. Yet the unitarity condition Eq.(103) forces this contribution to be antiparallel to $V^*(ud)V(us)$; i.e., again no relative phase.

The situation changes fundamentally for three families: the weak parameters $V(ij)$ now form a 3×3 matrix and the condition stated in Eq.(103) gets extended:

$$V(ud)V^*(us) + V(cd)V^*(cs) + V(td)V^*(ts) = 0 \tag{105}$$

This is a triangle relation in the complex plane. There emerge now relative phases between the weak parameters and the loop diagrams with internal charm and top quarks can generate CP asymmetries.

Unitarity imposes altogether nine algebraic conditions on the matrix elements of $\mathbf{V}_{CKM}$, of which six are triangle relations analogous to Eq.(105). There are several nice features about this representation in terms of triangles; I list four now and others later:

1. The *shape* of each triangle is independant of the phase convention adopted for the quark fields. Consider for example Eq.(105): changing the phase of any of the up-type quarks will not affect the triangle at all. Under $|s\rangle \to |s\rangle e^{i\phi_s}$ the whole triangle will rotate around the left end of its base line by an angle ϕ_s – yet the shape of the triangle – in contrast to its orientation in the complex plane – remains the same! The angles inside the triangles are thus observables; choosing an orientation for the triangles is then a matter of convenience.

2. It is easily shown that all six KM triangles possess the same area. Multiplying Eq.(105) by the phase factor $V^*(ud)V(us)/|V(ud)V(us)|$, which does not change the area, yields

$$|V(ud)V(us)| + \frac{V^*(ud)V(us)V(cd)V^*(cs)}{|V(ud)V(us)|} + \frac{V^*(ud)V(us)V(td)V^*(ts)}{|V(ud)V(us)|} = 0 \tag{106}$$

$$\text{area(triangle of Eq.(105))} = \frac{1}{2}|\text{Im}V(ud)V(cs)V^*(us)V^*(cd)| =$$
$$= \frac{1}{2}|\text{Im}V(ud)V(ts)V^*(us)V^*(td)| \tag{107}$$

Multiplying Eq.(105) instead by the phase factors $V^*(cd)V(cs)/|V(cd)V(cs)|$ or $V^*(td)V(ts)/|V(td)V(ts)|$ one sees that the area of this triangle can be expressed in other ways still. Among them is

$$\text{area(triangle of Eq.(105))} = \frac{1}{2}|\text{Im}V(cd)V(ts)V^*(cs)V^*(td)| \tag{108}$$

Due to the unitarity relation

$$V^*(cd)V(td) + V^*(cb)V(tb) = -V^*(cs)V(ts) \tag{109}$$

one has

$$\text{area(triangle of Eq.(105))} = \frac{1}{2}|\text{Im}V(cd)V(tb)V^*(cb)V^*(td)| \tag{110}$$

– yet this is exactly the area of the triangle defined by Eq.(109)! This is the re-incarnation of the original observation that there is a *single irreducible* weak phase for three families.

3. In general one has for the area of these triangles

$$A_{CPV}(\text{every triangle}) = \frac{1}{2}\text{J}$$

$$J = \text{Im}V^*(km)V(lm)V(kn)V^*(ln) = \text{Im}V^*(mk)V(ml)V(nk)V^*(nl) \quad (111)$$

irrespective of the indices k, l, m, n; J is obviously re-phasing invariant.

4. If there is a representation of V_{CKM} where all phases were confined to a 2×2 sub-matrix exactly rather than approximately, then one can rotate all these phases away; i.e., CP is conserved in such a scenario! Consider again the triangle described by Eq.(105): it can always be rotated such that its baseline – $V(ud)V^*(us)$ – is real. Then Im$V(td)V^*(ts)$ = - Im$V(cd)V^*(cs)$ holds. If, for example, there were no phases in the third row and column, one would have Im$(V(td)V^*(ts)) = 0$ and therefore Im$V(cd)V^*(cs) = 0$ as well; i.e., $V(ud)V^*(us)$ and $V(cd)V^*(cs)$ were real relative to each other; therefore $J = 0$, i.e. all six triangles had zero area meaning there are no relative weak phases!

2.8 Evaluating ϵ_K and ϵ'

In calculating observables in a given theory – in the case under study ϵ_K and ϵ' within the KM Ansatz – one is faced with the 'Dichotomy of the Two Worlds', namely

- one world of *short*-distance physics where even the strong interactions can be treated *perturbatively* in terms of quarks and gluons and in which theorists like to work, and
- the other world of *long*-distance physics where one has to deal with hadrons the behaviour of which is controlled by *non*-perturbative dynamics and where, by the way, everyone, including theorists, lives.

Accordingly the calculational task is divided into two parts, namely first determing the relevant transition operators in the short-distance world and then evaluating their matrix elements in the hadronic world.

$\Delta S = 2$ **Transitions**

Since the *elementary* interactions in the Standard Model can change strangeness at most by one unit, the $\Delta S = 2$ amplitude driving $K^0 - \bar{K}^0$ oscillations is obtained by iterating the basic $\Delta S = 1$ coupling:

$$\mathcal{L}_{eff}(\Delta S = 2) = \mathcal{L}(\Delta S = 1) \otimes \mathcal{L}(\Delta S = 1) \quad (112)$$

There are actually two ways in which the $\Delta S = 1$ transition can be iterated:

(A) The resulting $\Delta S = 2$ transition is described by a *local* operator. The celebrated box diagram makes this connection quite transparent. The contributions that do *not* depend on the mass of the internal quarks cancel against each other due

to the GIM mechanism. Integrating over the internal fields, namely the W bosons and the top and charm quarks [h] then yields a convergent result:

$$\mathcal{L}_{eff}^{box}(\Delta S = 2, \mu) = \left(\frac{G_F}{4\pi}\right)^2 \cdot$$

$$\left[\xi_c^2 E(x_c)\eta_{cc} + \xi_t^2 E(x_t)\eta_{tt} + 2\xi_c\xi_t E(x_c, x_t)\eta_{ct}\right] \left[\alpha_S(\mu^2)\right]^{-\frac{6}{27}} \left(\bar{d}\gamma_\mu(1-\gamma_5)s\right)^2 + h.c. \tag{113}$$

with ξ_i denoting combinations of KM parameters

$$\xi_i = V(is)V^*(id)\ , \quad i = c, t\ ; \tag{114}$$

$E(x_i)$ and $E(x_c, x_t)$ reflect the box loops with equal and different internal quarks, respectively [17]:

$$E(x_i) = x_i \left(\frac{1}{4} + \frac{9}{4(1-x_i)} - \frac{3}{2(1-x_i)^2}\right) - \frac{3}{2}\left(\frac{x_i}{1-x_i}\right)^3 \log x_i \tag{115}$$

$$E(x_c, x_t) = x_c x_t \left[\left(\frac{1}{4} + \frac{3}{2(1-x_t)} - \frac{3}{4(1-x_t)^2}\right)\frac{\log x_t}{x_t - x_c} + (x_c \leftrightarrow x_t) - \right.$$
$$\left. - \frac{3}{4}\frac{1}{(1-x_c)(1-x_t)}\right] \tag{116}$$

$$x_i = \frac{m_i^2}{M_W^2} \tag{117}$$

and η_{ij} containing the QCD radiative corrections from evolving the effective Lagrangian from M_W down to the internal quark mass. The factor $[\alpha_S(\mu^2)]^{-6/27}$ reflects the fact that a scale μ must be introduced at which the four-quark operator $(\bar{s}\gamma_\mu(1-\gamma_5)d)^2$ is defined. This dependance on the auxiliary variable μ drops out when one takes the matrix element of this operator (at least when one does it correctly). Including next-to-leading log corrections one finds (for $m_t \simeq 180$ GeV) [18]:

$$\eta_{cc} \simeq 1.38 \pm 0.20\ , \quad \eta_{tt} \simeq 0.57 \pm 0.01\ , \quad \eta_{cc} \simeq 0.47 \pm 0.04 \tag{118}$$

(B) However there is also a *non*-local $\Delta S = 2$ operator generated from the iteration of $\mathcal{L}(\Delta S = 1)$. While it presumably provides a major contribution to Δm_K, it is not sizeable for ϵ_K within the KM ansatz [i] and will be ignored here.

Even for a local four-fermion operator it is non-trivial to evaluate an on-shell matrix element between hadron states since that is clearly controlled by non-perturbative dynamics. Usually one parametrizes this matrix element as follows (in the phase convention $\mathbf{CP}|K^0\rangle = |\bar{K}^0\rangle$):

$$\langle K^0|(\bar{d}\gamma_\mu(1-\gamma_5)s)(\bar{d}\gamma_\mu(1-\gamma_5)s)|\bar{K}^0\rangle =$$

$$= \frac{4}{3}B_K\langle K^0|(\bar{d}\gamma_\mu(1-\gamma_5)s)|0\rangle\langle 0|(\bar{d}\gamma_\mu(1-\gamma_5)s)|\bar{K}^0\rangle = -\frac{4}{3}B_K f_K^2 m_K \tag{119}$$

[h] The up quarks act merely as a subtraction term here.
[i] This can be inferred from the observation that $|\epsilon'/\epsilon_K| \ll 0.05$

Method	B_K
Large N_C Expansion	$\frac{3}{4}$
Large N_C Chiral Pert. with loop correction	0.66 ± 0.1
Lattice QCD	0.84 ± 0.2

Table 1: Values of B_K from various theoretical techniques

The factor B_K is – for historical reasons of no consequence now – often called the bag factor; $B_K = 1$ is referred to as *vacuum saturation* or *factorization ansatz* since it corresponds to a situation where inserting the vacuum intermediate state into Eq.(119) reproduces the full result after all colour contractions of the quark lines have been included. Several theoretical techniques have been employed to estimate the size of B_K; their findings are listed in Table 1. These results, which are all consistent with each other and with several phenomenological studies as well, can be summarized as follows:

$$B_K \simeq 0.8 \pm 0.2 > 0 \tag{120}$$

Since the size of this matrix element is determined by the strong interactions, one indeed expects $B_K \sim 1$.

We have assembled all the ingredients now for calculating ϵ_K. The starting point is given by [j]

$$|\epsilon_K| \simeq \frac{1}{\sqrt{2}} \left| \frac{\mathrm{Im} M_{12}}{\Delta m_K} \right| \tag{121}$$

The CP-odd part $\mathrm{Im}M_{12}$ is obtained from

$$\mathrm{Im} M_{12} = \mathrm{Im} \langle K^0 | \mathcal{L}_{eff}(\Delta S = 2) | \bar{K}^0 \rangle \tag{122}$$

whereas for Δm_K one inserts the experimental value, since the long-distance contributions to Δm_K are not under theoretical control. One then finds

$$|\epsilon_K|_{KM} \simeq |\epsilon_K|^{box}_{KM} \simeq$$

$$\simeq \frac{G_F^2}{6\sqrt{2}\pi^2} \frac{M_W^2 m_K f_K^2 B_K}{\Delta m_K} \left[\mathrm{Im}\xi_c^2 E(x_c)\eta_{cc} + \mathrm{Im}\xi_t^2 E(x_t)\eta_{tt} + 2\mathrm{Im}(\xi_c\xi_t) E(x_c, x_t)\eta_{ct} \right]$$

$$\simeq 1.9 \cdot 10^4 B_K \left[\mathrm{Im}\xi_c^2 E(x_c)\eta_{cc} + \mathrm{Im}\xi_t^2 E(x_t)\eta_{tt} + 2\mathrm{Im}(\xi_c\xi_t) E(x_c, x_t)\eta_{ct} \right] \simeq$$

$$\simeq 7.8 \cdot 10^{-3} \eta B_K (1.4 - \rho) \tag{123}$$

where I have used the numerical values for the KM parameters listed above and $x_t \simeq 5$ corresponding to $m_t = 180$ GeV.

To reproduce the observed value of $|\epsilon_K|$ one needs

$$\eta \simeq \frac{0.3}{B_K} \frac{1}{1.4 - \rho} \tag{124}$$

[j]The exact expression is $|\epsilon_K| = \frac{1}{\sqrt{2}} \left| \frac{\mathrm{Im}M_{12}}{\Delta m_K} - \xi_0 \right|$ where ξ_0 denotes the phase of the $K^0 \to (\pi\pi)_{I=0}$ isospin zero amplitude; its contribution is numerically irrelevant.

For a given B_K one thus obtains another $\rho-\eta$ constraint. Since B_K is not precisely known one has a fairly broad band in the $\rho-\eta$ plane rather than a line. Yet I find it quite remarkable and very non-trivial that Eq.(124) *can* be satisfied since

$$\frac{0.3}{B_K} \sim 0.3 \div 0.5 \tag{125}$$

without stretching any of the parameters or bounds, in particular

$$\sqrt{\rho^2+\eta^2} \sim 0.38 \pm 0.11 \ . \tag{126}$$

While this does of course not amount to a *pre*diction, one should keep in mind for proper perspective that in the 1970's and early 1980's values like $|V(cb)| \sim 0.04$ and $|V(ub)| \sim 0.004$ would have seemed quite unnatural; claiming that the top quark mass had to be 180 GeV would have been outright preposterous even in the 1980's! Consider a scenario with $|V(cb)| \simeq 0.04$ and $|V(ub)| \simeq 0.003$, yet $m_t \simeq 40$ GeV; in the mid 80's this would have appeared to be quite natural (and there had even been claims that top quarks with a mass of 40 ± 10 GeV had been discovered). In that case one would need

$$\eta \sim \frac{0.75}{B_K} \tag{127}$$

to reproduce $|\epsilon_K|$. Such a large value for η would hardly be compatible with what we know about $|V(ub)|$.

♠ ♠ ♠

Homework Problem # 2:

Eq.(123) suggests that a non-vanishing value for ϵ_K is generated from the box diagram with internal charm quarks only – $\mathrm{Im}\xi_c^2 \, E(x_c) = -\eta A^2 \lambda^6 E(x_c) \neq 0$ – *without* top quarks. How does this match up with the statement that the intervention of three families is needed for a CP asymmetry to arise?

♠ ♠ ♠

An Aside on Δm_K (& $\Delta\Gamma_K$)

Having the local operator $\mathcal{L}^{box}_{\Delta S=2}$ generated by the box diagram, one can calculate its contribution to Δm_K, including the latter's sign, as expressed in Eq.(47). One finds

$$\frac{\Delta m_K|_{box}}{\Delta m_K|_{exp}} \sim 0.5 - 1 \ ; \tag{128}$$

i.e., the box diagram yields a major or even the dominant contribution to the observed value of Δm_K and it predicts indeed that the CP odd state is heavier, albeit by a whisker only. This is due to the following factors:

- The coefficient of the effective $\Delta S = 2$ operator is positive.
- The quantity B_K is reliably estimated to be positive, see Eq.(120).
- The minus sign in Eq.(119) thus cancels against that in Eq.(47) to yield $M_B > M_A$; the state K_B is the CP odd one in the convention of Eq.(22) which was also employed in obtaining Eq.(47).

There are sizeable long-distance contributions to Δm_K which are estimated to be positive, as well.

There is no reason why the box diagram's contribution to $\Delta\Gamma_K$ should have anything to do with reality: it is dominated by $K_L \to \pi\pi$ which cannot be described by short-distance dynamics. Nevertheless it is amusing to note that one finds $\Delta\Gamma_K^{box} > 0$ – in agreement with reality!

$\Delta S = 1$ Decays

At first one might think that no *direct* CP asymmetry can arise in $K \to \pi\pi$ decays since it requires the interplay of three quark families. Yet upon further reflection one realizes that a one-loop diagram produces the so-called Penguin operator which changes isopin by half a unit only, is *local* and contains a CP *odd* component since it involves virtual charm and top quarks. With direct CP violation thus being of order $\hbar$, i.e. a pure quantum effect, one suspects already at this point that it will be reduced in strength.

The quantity ϵ' is suppressed relative to ϵ_K due to two other reasons:

- The GIM factors are actually quite different for ϵ_K and ϵ': in the former case they are of the type $(m_t^2 - m_c^2)/M_W^2$, in the latter $\log(m_t^2/m_c^2)$. Both of these expressions vanish for $m_t = m_c$, yet for the realistic case $m_t \gg m_c$ they behave very differently: ϵ_K is much more enhanced by the large top mass than ϵ'. This means of course that $|\epsilon'/\epsilon_K|$ is a rather steeply decreasing function of m_t.

- There are actually two classes of Penguin operators contributing to ϵ', namely strong as well as electroweak Penguins. The latter become relevant since they are more enhanced than the former for very heavy top masses due to the coupling of the longitudinal virtual Z boson (the re-incarnation of one of the original Higgs fields) to the internal top line. Yet electroweak and strong Penguins contribute with the opposite sign!

CPT invariance together with the measured $\pi\pi$ phase shifts tells us that the two complex quantities ϵ' and ϵ_K are almost completely real to each other; i.e., their ratio is practically real:

$$\frac{\epsilon'}{\epsilon_K} \simeq 2\omega \frac{\Phi(\Delta S = 1)}{\Phi(\Delta S = 2)} \tag{129}$$

where, as defined before,

$$\omega \equiv \frac{|A_2|}{|A_0|} \simeq 0.05 \ , \ \ \Phi(\Delta S = 2) \equiv \arg \frac{M_{12}}{\Gamma_{12}} \ \ , \Phi(\Delta S = 1) \equiv \arg \frac{A_2}{A_0} \tag{130}$$

Eq.(129) makes two points obvious:

- Direct CP violation – $\epsilon' \neq 0$ – requires a relative phase between the isospin 0 and 2 amplitudes; i.e., $K \to (\pi\pi)_0$ and $K \to (\pi\pi)_2$ have to exhibit different CP properties.

- The observable ratio ϵ'/ϵ_K is *artifically reduced* by the enhancement of the $\Delta I = 1/2$ amplitude, as expressed through ω.

Several $\Delta S = 1$ transition operators contribute to ϵ' and their renormalization has to be treated quite carefully. Two recent detailed analyses yield [19,20]

$$-2.1 \cdot 10^{-4} \leq \frac{\epsilon'}{\epsilon_K} \leq 13.3 \cdot 10^{-4} \tag{131}$$

$$\frac{\epsilon'}{\epsilon_K} = (4.6 \pm 3.0 \pm 0.4) \cdot 10^{-4} \tag{132}$$

These results are quite consistent with each other and show

- that the KM ansatz leads to a prediction typically in the range *below* 10^{-3},
- that the value could happen to be zero or even slightly negative and
- that large theoretical uncertainties persist due to cancellations among various contributions.

This last (unfortunate) point can be illustrated also by comparing these predictions with older ones made before top quarks were discovered and their mass measured; those old predictions [21] are very similar to Eqs.(131,132), once the now known value of m_t has been inserted.

Two new experiments running now – NA 48 at CERN and KTEV at FNAL – and one expected to start up soon – CLOE at DAΦNE – expect to measure ϵ'/ϵ_K with a sensitivity of $\simeq \pm 2 \cdot 10^{-4}$. Concerning their future results one can distinguish four scenarios:

1. The 'best' scenario: $\epsilon'/\epsilon_K \geq 2 \cdot 10^{-3}$. One would then have established unequivocally direct CP violation of a strength that very probably reflects the intervention of new physics beyond the KM ansatz.
2. The 'tantalizing' scenario: $1 \cdot 10^{-3} \leq \epsilon'/\epsilon_K \leq 2 \cdot 10^{-3}$. It would be tempting to interprete this discovery of direct CP violation as a sign for new physics – yet one could not be sure!
3. The 'conservative' scenario: $\epsilon'/\epsilon_K \simeq \text{few} \cdot 10^{-4} > 0$. This strength of direct CP violation could easily be accommodated within the KM ansatz – yet no further constraint would materialize.
4. The 'frustrating' scenario: $\epsilon'/\epsilon_K \simeq 0$ within errors! No substantial conclusion could be drawn then concerning the presence or absence of direct CP violation, and the allowed KM parameter space would hardly shrink.

3 'Exotica'

In this section I will discuss important possible manifestations of CP and/or T violation that are exotic only in the sense that they are unobservably small with the KM ansatz, to be introduced in the next section.

$K_{3\mu}$ Decays In the reaction

$$K^+ \to \mu^+ \nu \pi^0 \tag{133}$$

one can search for a transverse polarisation of the emerging muons in close analogy to what has just been discussed for $K_L \to \pi^+\pi^-\gamma$:

$$P_\perp^{K^+}(\mu) \equiv \langle \vec{s}(\mu) \cdot (\vec{p}(\mu) \times \vec{p}(\pi^0)) \rangle \tag{134}$$

where $\vec{s}$ and $\vec{p}$ denote spin and momentum, respectively. The quantity $P_\perp(\mu)$ constitutes a T-odd correlation:

$$\left.\begin{array}{l} \vec{p} \Rightarrow -\vec{p} \\ \vec{s} \overset{\mathbf{T}}{\Rightarrow} -\vec{s} \end{array}\right\} \rightsquigarrow P_\perp(\mu) \overset{T}{\Rightarrow} -P_\perp(\mu) \tag{135}$$

Once a *non*-vanishing value has been observed for a parity-odd correlation one has unequivocally found a manifestation of parity violation. From $P_\perp^{K^+}(\mu) \neq 0$ one can deduce that T is violated – yet the argument is more subtle as can be learnt from the following homework problem.

♠ ♠ ♠

Homework Problem #3:

Consider

$$K_L \to \mu^+ \nu \pi^- \tag{136}$$

Does $P_\perp^{K_L}(\mu) \equiv \langle \vec{s}(\mu) \cdot (\vec{p}(\mu) \times \vec{p}(\pi^-)) \rangle \neq 0$ necessarily imply that T invariance does not hold in this reaction?

♠ ♠ ♠

Data on $P_\perp^{K^+}(\mu)$ are still consistent with zero [22]:

$$P_\perp^{K^+}(\mu) = (-1.85 \pm 3.60) \cdot 10^{-3} \; ; \tag{137}$$

yet being published in 1981 they are ancient by the standards of our disciplin.

On general grounds one infers that

$$P_\perp^{K^+}(\mu) \propto \mathrm{Im} \frac{f_-^*}{f_+} \tag{138}$$

holds where f_- [f_+] denotes the chirality changing [conserving] decay amplitude. Since f_- practically vanishes within the Standard Model, one obtains a fortiori $P_\perp^{K^+}(\mu)|_{KM} \simeq 0$.

Yet in the presence of charged Higgs fields one has $f_- \neq 0$. CPT implies that $P_\perp^{K^+}(\mu) \neq 0$ represents CP violation as well, and actually one of the *direct* variety. A rather model independant guestimate on how large such an effect could be is obtained from the present bound on ϵ'/ϵ_K:

$$P_\perp^{K^+}(\mu) \leq 20 \cdot (\epsilon'/\epsilon_K) \cdot \epsilon_K \leq 10^{-4} \tag{139}$$

where the factor 20 allows for the 'accidental' reduction of ϵ'/ϵ_K by the $\Delta I = 1/2$ rule: $\omega \simeq 1/20$. This bound is a factor of 100 larger than what one could obtain

within KM. It could actually be bigger still since there is a loophole in this generic argument: Higgs couplings to leptons could be strongly enhanced through a large ratio of vacuum expectation values v_1 relative to v_3, where v_1 controls the couplings to up-type quarks and v_3 to leptons. Then

$$P_\perp^{K^+}(\mu)|_{Higgs} \leq \mathcal{O}(10^{-3}) \tag{140}$$

becomes conceivable with the Higgs fields as heavy as 80 - 200 GeV [23]. Such Higgs exchanges would be quite insignificant for $K_L \to \pi\pi$!

Since $K_{\mu 3}$ studies provide such a unique window onto Higgs dynamics, I find it mandatory to probe for $P_\perp(\mu) \neq 0$ in a most determined way. It is gratifying to note that an on-going KEK experiment will be sensitive to $P_\perp(\mu)$ down to the 10^{-3} level – yet I strongly feel one should not stop there, but push further down to the 10^{-4} level.

3.1 Electric Dipole Moments

Consider a system – such as an elementary particle or an atom – in a weak external electric field $\vec{E}$. The energy shift of this system due to the electric field can then be expressed through an expansion in powers of $\vec{E}$ [24]:

$$\Delta E = \vec{d} \cdot \vec{E} + d_{ij} E_i E_j + \mathcal{O}(|\vec{E}|^3) \tag{141}$$

where summation over the indices i, j is understood. The coefficient $\vec{d}$ of the term linear in $\vec{E}$ is called electric dipole moment or sometimes permanent electric dipole moment (hereafter referred to as EDM) whereas that of the quadratic term is often named an *induced* dipole moment.

For an elementary object one has

$$\vec{d} = d\vec{j} \tag{142}$$

where $\vec{j}$ denotes its total angular momentum since that is the only available vector. Under time reversal one finds

$$\begin{array}{l} \vec{j} \overset{\mathbf{T}}{\Rightarrow} -\vec{j} \\ \vec{E} \overset{\mathbf{T}}{\Rightarrow} \vec{E} \, . \end{array} \tag{143}$$

Therefore

$$\text{T invariance} \rightsquigarrow d = 0 \; ; \tag{144}$$

i.e., such an electric dipole moment has to vanish, unless T is violated (and likewise for parity).

The EDM is at times confused with an induced electric dipole moment objects can possess due to their internal structure. To illustrate that consider an atom with two *nearly degenerate* states of opposite parity:

$$\mathbf{P}|\pm\rangle = \pm|\pm\rangle \; , \; \mathbf{H}|\pm\rangle = E_\pm|\pm\rangle \; , \; E_+ < E_- \; , \; \frac{E_- - E_+}{E_+} \ll 1 \tag{145}$$

Placed in a constant external electric field $\vec{E}$ the states $|\pm\rangle$ will mix to produce new energy eigenstates; those can be found by diagonalising the matrix of the Hamilton operator:

$$H = \begin{pmatrix} E_+ & \Delta \\ \Delta & E_- \end{pmatrix} \tag{146}$$

where $\Delta = \vec{d}_{ind} \cdot \vec{E}$ with $\vec{d}_{ind}$ being the transition matrix element between the $|+\rangle$ and $|-\rangle$ states induced by the electric field. The two new energy eigenvalues are

$$E_{1,2} = \frac{1}{2}(E_+ + E_-) \pm \sqrt{\frac{1}{4}(E_+ - E_-)^2 + \Delta^2} \tag{147}$$

For $E_+ \simeq E_-$ one has

$$E_{1,2} \simeq \frac{1}{2}(E_+ + E_-) \pm |\Delta| \; ; \tag{148}$$

i.e., the energy shift appears to be linear in $\vec{E}$:

$$\Delta E = E_2 - E_1 = 2|\vec{d}_{ind} \cdot \vec{E}| \tag{149}$$

Yet with $\vec{E}$ being sufficiently small one arrives at $4(\vec{d}_{ind} \cdot \vec{E})^2 \ll (E_+ - E_-)^2$ and therefore

$$E_1 \simeq E_- + \frac{(\vec{d}_{ind} \cdot \vec{E})^2}{E_- - E_+} \; , \; E_2 \simeq E_+ - \frac{(\vec{d}_{ind} \cdot \vec{E})^2}{E_- - E_+} \; ; \tag{150}$$

i.e., the induced energy shift is *quadratic* in $\vec{E}$ rather than linear and therefore does *not* imply T violation! The distinction between an EDM and an induced electric dipole moment is somewhat subtle – yet it can be established in an unequivocal way by probing for a linear Stark effect with weak electric fields. A more careful look at Eq.(149) already indicates that. For the energy shift stated there does not change under $\vec{E} \Rightarrow -\vec{E}$ as it should for an EDM which also violates parity!

The data for neutrons read:

$$d_n = \begin{cases} (-3 \pm 5) \cdot 10^{-26} \; ecm & \text{ILL} \\ (2.6 \pm 4 \pm 1.6) \cdot 10^{-26} \; ecm & \text{LNPI} \end{cases} \tag{151}$$

These numbers and the experiments leading to them are very impressive:

- One uses neutrons emanating from a reactor and subsequently cooled down to a temperature of order 10^{-7} eV. This is comparable to the kinetic energy a neutron gains when dropping 1 m in the earth's gravitational field.
- Extrapolating the ratio between the neutron's radius – $r_N \sim 10^{-13}$ cm – with its EDM of no more than 10^{-25} ecm to the earth's case, one would say that it corresponds to a situation where one has searched for a displacement in the earth's mass distribution of order $10^{-12} \cdot r_{earth} \sim 10^{-3}$cm = 10 microns!

A truly dramatic increase in sensitivity for the *electron's* EDM has been achieved over the last few years:

$$d_e = (-0.3 \pm 0.8) \cdot 10^{-26} \;\; e\,cm \tag{152}$$

This quantity is searched for through measuring electric dipole moments of *atoms*. At first this would seem to be a losing proposition theoretically: for according to Schiff's theorem an atom when placed inside an external electric field gets deformed in such a way that the electron's EDM is completely shielded; i.e., $d_{atom} = 0$. This theorem holds true in the nonrelativistic limit, yet is vitiated by relativistic effects. Not surprisingly the latter are particularly large for heavy atoms; one would then expect the electron's EDM to be only partially shielded: $d_{atom} = S \cdot d_e$ with $S < 1$. Yet amazingly – and highly welcome of course – the electron's EDM can actually get magnified by two to three orders of magnitude in the atom's electric dipole moment; for Caesium one has [24]

$$d_{Cs} \simeq 100 \cdot d_e \tag{153}$$

This enhancement factor is the theoretical reason behind the greatly improved sensitivity for d_e as expressed through Eq.(152); the other one is experimental, namely the great strides made by laser technology applied to atomic physics.

The quality of the number in Eq.(152) can be illustrated through a comparison with the electron's magnetic moment. The electromagnetic form factor $\Gamma_\mu(q)$ of a particle like the electron evaluated at momentum transfer q contains two tensor terms:

$$d_{atom} = \frac{1}{2m_e}\sigma_{\mu\nu}q^\nu \left[iF_2(q^2) + F_3(q^2)\gamma_5\right] + ... \tag{154}$$

In the nonrelativistic limit one finds for the EDM:

$$d_e = -\frac{1}{2m_e}F_3(0) \tag{155}$$

On the other hand one has

$$\frac{1}{2}(g-2) = \frac{1}{e}F_2(0) \tag{156}$$

The *precision* with which $g-2$ is known for the electron – $\delta[(g-2)/2] \simeq 10^{-11}$ – (and which represents one of the great success stories of field theory) corresponds to an *uncertainty* in the electron's *magnetic* moment

$$\delta\left[\frac{1}{2m_e}F_2(0)\right] \simeq 2 \cdot 10^{-22} \;\; e\,cm \tag{157}$$

that is several orders of magnitude larger than the bound on its EDM!

Since the EDM is, as already indicated above, described by a dimension-five operator in the Lagrangian

$$\mathcal{L}_{EDM} = -\frac{i}{2}d\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi F^{\mu\nu} \tag{158}$$

with $F^{\mu\nu}$ denoting the electromagnetic field strength tensor, one can calculate d within a given theory of CP violation as a finite quantity. Within the KM ansatz one finds that the neutron's EDM is zero for all practical purposes [k]:

$$d_N|_{KM} < 10^{-30} \;\; e\,cm \tag{159}$$

[k] I ignore here the Strong CP Problem.

and likewise for d_e. Yet again that is due to very specific features of the KM mechanism and the chirality structure of the Standard Model. In alternative models – where CP violation enters through *right*-handed currents or a non-minimal Higgs sector (with or without involving SUSY) – one finds

$$d_N|_{New\ Physics} \sim 10^{-27} - 10^{-28} \quad e\,cm \tag{160}$$

as reasonable benchmark figures.

4 Summary on the CP Phenomenology with Light Degrees of Freedom

To summarize our discussion up to this point:

- The following data represent the most sensitive probes:

$$\mathrm{BR}(K_L \to \pi^+\pi^-) = 2.3 \cdot 10^{-3} \neq 0 \tag{161}$$

$$\frac{\mathrm{BR}(K_L \to l^+\nu\pi^-)}{\mathrm{BR}(K_L \to l^-\nu\pi^+)} \simeq 1.006 \neq 1 \tag{162}$$

$$\mathrm{Re}\frac{\epsilon'}{\epsilon_K} = \begin{cases} (2.3 \pm 0.7) \cdot 10^{-3} & NA\,31 \\ (0.6 \pm 0.58 \pm 0.32 \pm 0.18) \cdot 10^{-3} & E\,731 \end{cases} \tag{163}$$

$$\mathrm{Pol}_\perp^{K^+}(\mu) = (-1.85 \pm 3.60) \cdot 10^{-3} \tag{164}$$

$$d_N < 12 \cdot 10^{-26} \quad e\,cm \tag{165}$$

$$d_{Tl} = (1.6 \pm 5.0) \cdot 10^{-24} \quad e\,cm \stackrel{theor.}{\rightsquigarrow} d_e = (-2.7 \pm 8.3) \cdot 10^{-27} \quad e\,cm \tag{166}$$

- An impressive amount of experimental ingenuity, acumen and commitment went into producing this list. We know that CP violation unequivocally exists in nature; it can be characterized by a *single* non-vanishing quantity:

$$\mathrm{Im}M_{12} \simeq 1.1 \cdot 10^{-8}\, eV \neq 0 \tag{167}$$

- The 'Superweak Model' states that there just happens to exist a $\Delta S = 2$ interaction that is fundamental or effective − whatever the case may be − generating $\mathrm{Im}M_{12} = \mathrm{Im}M_{12}|_{exp}$ while $\epsilon' = 0$. It provides merely a *classification* for possible dynamical implementations rather than such a dynamical implementation itself.

- The KM ansatz allows us to incorporate CP violation into the Standard Model. Yet it does not regale us with an understanding. Instead it relates the origins of CP violation to central mysteries of the Standard Model: Why are there families? Why are there three of those? What is underlying the observed pattern in the fermion masses?

- Still the KM ansatz succeeds in *accommodating* the data in an unforced way: ϵ_K emerges to be naturally small, ϵ' naturally tiny (once the huge top mass is built in), the EDM's for neutrons [electrons] naturally (tiny)^2 [(tiny)^3] etc.

5 CP Violation in Beauty Decays – The KM Perspective

The KM predictions for strange decays and electric dipole moments given above will be subjected to sensitive tests in the foreseeable future. Yet there is one question that most naturally will come up in this context: "Where else to look?" I will show below that on very general grounds one has to conclude that the decays of beauty hadrons provide by far the optimal lab. Yet first I want to make some historical remarks.

5.1 The Emerging Beauty of B Hadrons

Persistence Awarded!

In 1970 Lederman's group studying the Drell-Yan process

$$pp \to \mu^+\mu^- X \tag{168}$$

at Brookhaven observed a shoulder in the di-muon mass distribution around 3 GeV. 1974 saw the 'Octobre Revolution' when Ting et al. and Richter et al. found a narrow resonance – the J/ψ – with a mass of 3.1 GeV at Brookhaven and SLAC, respectively, and announced it. In 1977 Lederman's group working at Fermilab discovered three resonances in the mass range of 9.5 - 10.3 GeV, the Υ, Υ' and Υ''! That shows that persistence can pay off – at least sometimes and for some people.

Longevity of Beauty

The lifetime of weakly decaying beauty quarks can be related to the muon lifetime

$$\tau(b) \sim \tau(\mu) \left(\frac{m_\mu}{m_b}\right)^5 \frac{1}{9} \frac{1}{|V(cb)|^2} \sim 3 \cdot 10^{-14} \left|\frac{\sin\theta_C}{V(cb)}\right|^2 \text{ sec} \tag{169}$$

for a b quark mass of around 5 GeV; the factor 1/9 reflects the fact that the virtual W^- boson in b quark decays can materialize as a $d\bar{u}$ or $s\bar{c}$ in three colours each and as three lepton pairs. I have ignored phase space corrections here. Since the b quark has to decay outside its own family one would expect $|V(cb)| \sim \mathcal{O}(\sin\theta_C) = |V(us)|$. Yet starting in 1982 data showed a considerably longer lifetime

$$\tau(\text{beauty}) \sim 10^{-12} \text{ sec} \tag{170}$$

implying

$$|V(cb)| \sim \mathcal{O}(\sin^2\theta_C) \sim 0.05 \tag{171}$$

The technology to resolve decay vertices for objects of such lifetimes happened to have just been developed – for charm studies!

The Changing Identity of Neutral B Mesons

Speedy $B_d - \bar{B}_d$ oscillations were discovered by ARGUS in 1986:

$$x_d \equiv \frac{\Delta m(B_d)}{\Gamma(B_d)} \simeq \mathcal{O}(1) \tag{172}$$

These oscillations can then be tracked like the decays. This observation was also the first evidence that top quarks had to be heavier than orginally thought, namely $m_t \geq M_W$.

Beauty Goes to Charm (almost always)

It was soon found that b quarks exhibit a strong preference to decay into charm rather than up quarks

$$\left|\frac{V(ub)}{V(cb)}\right|^2 \ll 1 \tag{173}$$

establishing thus the hierarchy

$$|V(ub)|^2 \ll |V(cb)|^2 \ll |V(us)|^2 \ll 1 \tag{174}$$

Resume

We will soon see how all these observations form crucial inputs to the general message that big CP asymmetries should emerge in B decays and that they (together with interesting rare decays) are within reach of experiments. It is for this reason that I strongly feel that the only appropriate name for this quantum number is *beauty*! A name like bottom would not do it justice.

5.2 The KM Paradigm of Huge CP Asymmetries

Large Weak Phases!

The Wolfenstein representation expresses the CKM matrix as an expansion:

$$V_{CKM} = \begin{pmatrix} 1 & \mathcal{O}(\lambda) & \mathcal{O}(\lambda^3) \\ \mathcal{O}(\lambda) & 1 & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & 1 \end{pmatrix} \quad , \quad \lambda = \sin\theta_C \tag{175}$$

The crucial element in making this expansion meaningful is the 'long' lifetime of beauty hadrons of around 1 psec. That number had to change by an order of magnitude – which is out of the question – to invalidate the conclusions given below for the size of the weak phases.

The unitarity condition yields 6 triangle relations:

$$\underset{\mathcal{O}(\lambda)}{V^*(ud)V(us)} + \underset{\mathcal{O}(\lambda)}{V^*(cd)V(cs)} + \underset{\mathcal{O}(\lambda^5)}{V^*(td)V(ts)} = \delta_{ds} = 0 \tag{176}$$

$$\underset{\mathcal{O}(\lambda)}{V^*(ud)V(cd)} + \underset{\mathcal{O}(\lambda)}{V^*(us)V(cs)} + \underset{\mathcal{O}(\lambda^5)}{V^*(ub)V(cb)} = \delta_{uc} = 0 \tag{177}$$

$$\underset{\mathcal{O}(\lambda^4)}{V^*(us)V(ub)} + \underset{\mathcal{O}(\lambda^2)}{V^*(cs)V(cb)} + \underset{\mathcal{O}(\lambda^2)}{V^*(ts)V(tb)} = \delta_{sb} = 0 \tag{178}$$

$$\underset{\mathcal{O}(\lambda^4)}{V^*(td)V(cd)} + \underset{\mathcal{O}(\lambda^2)}{V^*(ts)V(cs)} + \underset{\mathcal{O}(\lambda^2)}{V^*(tb)V(cb)} = \delta_{ct} = 0 \tag{179}$$

$$\underset{\mathcal{O}(\lambda^3)}{V^*(ud)V(ub)} + \underset{\mathcal{O}(\lambda^3)}{V^*(cd)V(cb)} + \underset{\mathcal{O}(\lambda^3)}{V^*(td)V(tb)} = \delta_{db} = 0 \tag{180}$$

$$\underset{\mathcal{O}(\lambda^3)}{V^*(td)V(ud)} + \underset{\mathcal{O}(\lambda^3)}{V^*(ts)V(us)} + \underset{\mathcal{O}(\lambda^3)}{V^*(tb)V(ub)} = \delta_{ut} = 0 \tag{181}$$

where below each product of matrix elements I have noted their size in powers of λ.

We see that the six triangles fall into three categories:

1. The first two triangles are extremely 'squashed': two sides are of order λ, the third one of order λ^5 and their ratio of order $\lambda^4 \simeq 2.3 \cdot 10^{-3}$; Eq.(176) and Eq.(177) control the situation in strange and charm decays; the relevant weak phases there are obviously tiny.

2. The third and fourth triangles are still rather squashed, yet less so: two sides are of order λ^2 and the third one of order λ^4.

3. The last two triangles have sides that are all of the same order, namely λ^3. All their angles are therefore naturally large, i.e. $\sim$ several $\times$ 10 degrees! Since to leading order in λ one has

$$V(ud) \simeq V(tb)\ ,\ V(cd) \simeq -V(us)\ ,\ V(ts) \simeq -V(cb) \tag{182}$$

   we see that the triangles of Eqs.(180, 181) actually coincide to that order.

The sides of this triangle having naturally large angles are given by $\lambda \cdot V(cb)$, $V(ub)$ and $V^*(td)$; these are all quantities that control important aspects of B decays, namely CKM favoured and disfavoured B decays and $B_d - \bar{B}_d$ oscillations!

Different, Yet Coherent Amplitudes!

$B^0 - \bar{B}^0$ oscillations provide us with two different amplitudes that by their very nature have to be coherent:

$$B^0 \Rightarrow \bar{B}^0 \rightarrow f \leftarrow B^0 \tag{183}$$

On general grounds one expects oscillations to be speedy for $B^0 - \bar{B}^0$ (like for $K^0 - \bar{K}^0$), yet slow for $D^0 - \bar{D}^0$ [l]. Experimentally one indeed finds

$$\frac{\Delta m(B_d)}{\Gamma(B_d)} = 0.71 \pm 0.06 \tag{184}$$

$$\frac{\Delta m(B_s)}{\Gamma(B_s)} \geq 10 \tag{185}$$

While Eq.(184) describes an almost optimal situation the overly rapid pace of $B_s - \bar{B}_s$ oscillations will presumably cause experimental problems.

[l] $T^0 - \bar{T}^0$ oscillations cannot occur since top quarks decay before they hadronize [25].

The conditions are quite favourable also for *direct* CP violation to surface. Consider a transition amplitude

$$T(B \to f) = \mathcal{M}_1 + \mathcal{M}_2 = e^{i\phi_1} e^{i\alpha_1} |\mathcal{M}_1| + e^{i\phi_2} e^{i\alpha_2} |\mathcal{M}_2| \ . \tag{186}$$

The two partial amplitudes $\mathcal{M}_1$ and $\mathcal{M}_2$ are distinguished by, say, their isospin – as it was the case for $K \to (\pi\pi)_{I=0,2}$ discussed before; ϕ_1, ϕ_2 denote the phases in the *weak* couplings and α_1, α_2 the phase shifts due to *strong* final state interactions. For the CP conjugate reaction one obtains

$$T(\bar{B} \to \bar{f}) = e^{-i\phi_1} e^{i\alpha_1} |\mathcal{M}_1| + e^{-i\phi_2} e^{i\alpha_2} |\mathcal{M}_2| \ . \tag{187}$$

since under CP the weak parameters change into their complex conjugate values whereas the phase shifts remain the same; for the strong forces driving final state interactions conserve CP. The rate difference is then given by

$$\begin{aligned} \Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f}) \propto |T(B \to f)|^2 - |T(\bar{B} \to \bar{f})|^2 = \\ = -4\sin(\phi_1 - \phi_2) \cdot \sin(\alpha_1 - \alpha_2) \cdot \mathcal{M}_1 \otimes \mathcal{M}_2 \end{aligned} \tag{188}$$

For an asymmetry to arise in this way two conditions need to be satisfied simultaneously, namely

$$\begin{array}{c} \phi_1 \neq \phi_2 \\ \alpha_1 \neq \alpha_2 \end{array} \tag{189}$$

I.e., the two amplitudes $\mathcal{M}_1$ and $\mathcal{M}_2$ have to differ both in their weak and strong forces! The first condition implies (within the Standard Model) that the reaction has to be KM suppressed, whereas the second one require the intervention of nontrivial final state interactions.

There is a large number of KM suppressed channels in B decays that are suitable in this context: they receive significant contributions from weak couplings with large phases – like $V(ub)$ in the Wolfenstein representation – and there is no reason why the phase shifts should be small in general (although that could happen in some cases).

Resume

Let me summarize the discussion just given and anticipate the results to be presented below.

- *Large* CP asymmetries are *pre*dicted *with* confidence to occur in B decays. If they are not found, there is no plausible deniability for the KM ansatz.
- Some of these predictions can be made with high *parametric* reliability.
- New theoretical technologies have emerged that will allow us to translate this *parametric* reliability into *numerical* precision.
- Some of the observables exhibit a high and unambiguous sensitivity to the presence of New Physics since we are dealing with coherent processes with observables depending *linearly* on New Physics amplitudes and where the CKM 'background' is (or can be brought) under theoretical control.

5.3 General Phenomenology

Decay rates for CP conjugate channels can be expressed as follows:

$$\begin{aligned} \text{rate}(B(t) \to f) &= e^{-\Gamma_B t} G_f(t) \\ \text{rate}(\bar{B}(t) \to \bar{f}) &= e^{-\Gamma_B t} \bar{G}_{\bar{f}}(t) \end{aligned} \tag{190}$$

where CPT invariance has been invoked to assign the same lifetime Γ_B^{-1} to B and $\bar{B}$ hadrons. Obviously if

$$\frac{G_f(t)}{\bar{G}_{\bar{f}}(t)} \neq 1 \tag{191}$$

is observed, CP violation has been found. Yet one should keep in mind that this can manifest itself in two (or three) qualitatively different ways:

1. $$\frac{G_f(t)}{\bar{G}_{\bar{f}}(t)} \neq 1 \;\text{ with }\; \frac{d}{dt}\frac{G_f(t)}{\bar{G}_{\bar{f}}(t)} = 0\;; \tag{192}$$

   i.e., the *asymmetry* is the same for all times of decay. This is true for *direct* CP violation; yet, as explained later, it also holds for CP violation *in the oscillations*.

2. $$\frac{G_f(t)}{\bar{G}_{\bar{f}}(t)} \neq 1 \;\text{ with }\; \frac{d}{dt}\frac{G_f(t)}{\bar{G}_{\bar{f}}(t)} \neq 0\;; \tag{193}$$

   here the asymmetry varies as a function of the time of decay. This can be referred to as CP violation *involving oscillations*.

Quantum mechanics with its linear superposition principle makes very specific statements about the possible time dependance of $G_f(t)$ and $\bar{G}_{\bar{f}}(t)$; yet before going into that I want to pose another homework problem:

♠ ♠ ♠

Homework Problem # 4:

Consider the reaction

$$e^+e^- \to \phi \to (\pi^+\pi^-)_K(\pi^+\pi^-)_K \tag{194}$$

Its occurrance requires CP violation. For the *initial* state – ϕ – carries *even* CP parity whereas the *final* state with the two $(\pi^+\pi^-)$ combinations forming a P wave must be CP *odd*: $(+1)^2(-1)^l = -1$! Yet Bose statistics requiring identical states to be in a symmetric configuration would appear to veto this reaction; for it places the two $(\pi^+\pi^-)$ states into a P wave which is antisymmetric. What is the flaw in this reasoning? The same puzzle can be formulated in terms of

$$e^+e^- \to \Upsilon(4S) \to B_d\bar{B}_d \to (\psi K_S)_B(\psi K_S)_B\;. \tag{195}$$

♠ ♠ ♠

A straightforward application of quantum mechanics yields the general expressions [26,27,29]:

$$G_f(t) = |T_f|^2 \left[\left(1+\left|\tfrac{q}{p}\right|^2|\bar{\rho}_f|^2\right) + \left(1-\left|\tfrac{q}{p}\right|^2|\bar{\rho}_f|^2\right)\cos\Delta m_B t - 2(\sin\Delta m_B t)\mathrm{Im}\tfrac{q}{p}\bar{\rho}_f\right]$$
$$\bar{G}_{\bar{f}}(t) = |\bar{T}_{\bar{f}}|^2 \left[\left(1+\left|\tfrac{p}{q}\right|^2|\rho_{\bar{f}}|^2\right) + \left(1-\left|\tfrac{p}{q}\right|^2|\rho_{\bar{f}}|^2\right)\cos\Delta m_B t - 2(\sin\Delta m_B t)\mathrm{Im}\tfrac{p}{q}\rho_{\bar{f}}\right] \tag{196}$$

The amplitudes for the instantaneous $\Delta B = 1$ transition into a final state f are denoted by $T_f = T(B \to f)$ and $\bar{T}_f = T(\bar{B} \to f)$ and

$$\bar{\rho}_f = \frac{\bar{T}_f}{T_f} \;\;, \rho_{\bar{f}} = \frac{T_{\bar{f}}}{\bar{T}_{\bar{f}}} \;\;, \frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \tag{197}$$

Staring at the general expression is not always very illuminating; let us therefore consider three very simplified limiting cases:

- $\Delta m_B = 0$, i.e. *no* $B^0 - \bar{B}^0$ oscillations:

$$G_f(t) = 2|T_f|^2 \;\;,\;\; \bar{G}_{\bar{f}}(t) = 2|\bar{T}_{\bar{f}}|^2 \rightsquigarrow \frac{\bar{G}_{\bar{f}}(t)}{G_f(t)} = \left|\frac{\bar{T}_{\bar{f}}}{T_f}\right|^2 \;\;, \frac{d}{dt}G_f(t) \equiv 0 \equiv \frac{d}{dt}\bar{G}_{\bar{f}}(t) \tag{198}$$

  This is explicitely what was referred to above as *direct* CP violation.

- $\Delta m_B \neq 0$ and f a flavour-*specific* final state with *no* direct CP violation; i.e., $T_f = 0 = \bar{T}_{\bar{f}}$ and $\bar{T}_f = T_{\bar{f}}$:[m]

$$G_f(t) = \left|\tfrac{q}{p}\right|^2 |\bar{T}_f|^2(1-\cos\Delta m_B t) \;\;,\;\; \bar{G}_{\bar{f}}(t) = \left|\tfrac{p}{q}\right|^2 |T_{\bar{f}}|^2(1-\cos\Delta m_B t)$$
$$\rightsquigarrow \tfrac{\bar{G}_{\bar{f}}(t)}{G_f(t)} = \left|\tfrac{q}{p}\right|^4 \;\;,\;\; \tfrac{d}{dt}\tfrac{\bar{G}_{\bar{f}}(t)}{G_f(t)} \equiv 0 \;\;,\;\; \tfrac{d}{dt}\bar{G}_{\bar{f}}(t) \neq 0 \neq \tfrac{d}{dt}G_f(t) \tag{199}$$

  This constitutes CP violation *in the oscillations*. For the CP conserving decay into the flavour-specific final state is used merely to track the flavour identity of the decaying meson. This situation can therefore be denoted also in the following way:

$$\frac{\mathrm{Prob}(B^0 \Rightarrow \bar{B}^0;t) - \mathrm{Prob}(\bar{B}^0 \Rightarrow B^0;t)}{\mathrm{Prob}(B^0 \Rightarrow \bar{B}^0;t) + \mathrm{Prob}(\bar{B}^0 \Rightarrow B^0;t)} = \frac{|q/p|^2 - |p/q|^2}{|q/p|^2 + |p/q|^2} = \frac{1-|p/q|^4}{1+|p/q|^4} \tag{200}$$

- $\Delta m_B \neq 0$ with f now being a flavour-*non*specific final state – a final state *common* to B^0 and $\bar{B}^0$ decays – of a special nature, namely a CP eigenstate – $|\bar{f}\rangle = \mathbf{CP}|f\rangle = \pm|f\rangle$ – *without* direct CP violation – $|\bar{\rho}_f| = 1 = |\rho_{\bar{f}}|$:

$$G_f(t) = 2|T_f|^2 \left[1 - (\sin\Delta m_B t)\cdot \mathrm{Im}\tfrac{q}{p}\bar{\rho}_f\right]$$
$$\bar{G}_f(t) = 2|T_f|^2 \left[1 + (\sin\Delta m_B t)\cdot \mathrm{Im}\tfrac{q}{p}\bar{\rho}_f\right] \tag{201}$$
$$\rightsquigarrow \tfrac{d}{dt}\tfrac{\bar{G}_f(t)}{G_f(t)} \neq 0$$

---

[m] For a flavour-specific mode one has in general $T_f \cdot T_{\bar{f}} = 0$; the more intriguing case arises when one considers a transition that requires oscillations to take place.

is the concrete realization of what was called CP violation *involving oscillations*.

CP Violation in Oscillations

Using the convention blessed by the PDG

$$B = [\bar{b}q] \;\;,\;\; \bar{B} = [\bar{q}b] \tag{202}$$

we have

$$\begin{aligned} T(B \to l^- X) = 0 = T(\bar{B} \to l^+ X) \\ T_{SL} \equiv T(B \to l^+ X) = T(\bar{B} \to l^- X) \end{aligned} \tag{203}$$

with the last equality enforced by CPT invariance. The so-called Kabir test can then be realized as follows:

$$\frac{\mathrm{Prob}(B^0 \Rightarrow \bar{B}^0; t) - \mathrm{Prob}(\bar{B}^0 \Rightarrow B^0; t)}{\mathrm{Prob}(B^0 \Rightarrow \bar{B}^0; t) + \mathrm{Prob}(\bar{B}^0 \Rightarrow B^0; t)} =$$

$$= \frac{\mathrm{Prob}(B^0 \Rightarrow \bar{B}^0 \to l^- X; t) - \mathrm{Prob}(\bar{B}^0 \Rightarrow B^0 \to l^+ X; t)}{\mathrm{Prob}(B^0 \Rightarrow \bar{B}^0 \to l^- X; t) + \mathrm{Prob}(\bar{B}^0 \Rightarrow B^0 \to l^+ X; t)} = \frac{1 - |q/p|^4}{1 + |q/p|^4} \tag{204}$$

Without going into details I merely state the results here [29]:

$$1 - \left|\frac{q}{p}\right| \simeq \frac{1}{2}\mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) \sim \begin{cases} 10^{-3} & \text{for } B_d = (\bar{b}d) \\ 10^{-4} & \text{for } B_s = (\bar{b}s) \end{cases} \tag{205}$$

i.e.,

$$a_{SL}(B^0) \equiv \frac{\Gamma(\bar{B}^0(t) \to l^+ \nu X) - \Gamma(B^0(t) \to l^- \bar{\nu} X)}{\Gamma(\bar{B}^0(t) \to l^+ \nu X) + \Gamma(B^0(t) \to l^- \bar{\nu} X)} \simeq \begin{cases} \mathcal{O}(10^{-3}) & \text{for } B_d \\ \mathcal{O}(10^{-4}) & \text{for } B_s \end{cases} \tag{206}$$

The smallness of the quantity $1 - |q/p|$ is primarily due to $|\Gamma_{12}| \ll |M_{12}|$ or $\Delta\Gamma_B \ll \Delta m_B$. Within the Standard Model this hierarchy is understood (semi-quantitatively at leaast) as due to the hierarchy in the GIM factors of the box diagram expressions for Γ_{12} and M_{12}, namely $m_c^2/M_W^2 \ll m_t^2/M_W^2$.

For B_s mesons the phase between Γ_{12} and M_{12} is further (Cabibbo) suppressed for reasons that are peculiar to the KM ansatz: for to leading order in the KM parameters quarks of the second and third family only contribute and therefore $\arg(\Gamma_{12}/M_{12}) = 0$ to that order. If New Physics intervenes in $B^0 - \bar{B}^0$ oscillations, it would quite naturally generate a new phase between Γ_{12} and M_{12}; it could also reduce M_{12}. Altogether this CP asymmetry could get enhanced very considerably:

$$a_{SL}^{New\ Physics}(B^0) \sim 1\% \tag{207}$$

Therefore one would be ill-advised to accept the somewhat pessimistic KM predictions as gospel.

Since this CP asymmetry does not vary with the time of decay, a signal is not diluted by integrating over all times. It is, however, essential to 'flavour tag' the decaying meson; i.e., determine whether it was *produced* as a B^0 or $\bar{B}^0$. This can be achieved in several ways as discussed later.

Direct CP Violation

Sizeable direct CP asymmetries arise rather naturally in B decays. Consider

$$b \to s\bar{u}u \tag{208}$$

Three different processes contribute to it, namely

- the tree process
$$b \to uW^* \to u(\bar{u}s)_W \ , \tag{209}$$

- the penguin process with an internal top quark which is purely local (since $m_t > m_b$)
$$b \to sg^* \to su\bar{u} \ , \tag{210}$$

- the penguin reaction with an internal charm quark. Since $m_b > 2m_c + m_s$, this last operator is *not* local: it contains an absorptive part that amounts to a final state interaction including a phase shift.

One then arrives at a guestimate [28,29]

$$\frac{\Gamma(b \to su\bar{u}) - \Gamma(\bar{b} \to \bar{s}u\bar{u})}{\Gamma(b \to su\bar{u}) + \Gamma(\bar{b} \to \bar{s}u\bar{u})} \sim \mathcal{O}(\%) \tag{211}$$

Invoking quark-hadron duality one can expect (or at least hope) that this quark level analysis – rather than being washed out by hadronisation – yields some average asymmetry or describes the asymmetry for some inclusive subclass of nonleptonic channels. I would like to draw the following lessons from these considerations:

- According to the KM ansatz the natural scale for direct CP asymmetries in the decays of beauty hadrons (neutral or charged mesons or baryons) is the 10^{-2} level – not $10^{-6} \div 10^{-5}$ as in strange decays!

- The size of the asymmetry in *individual* channels – like $B \to K\pi$ – is shaped by the strong final state interactions operating there. Those are likely to differ considerably from channel to channel, and at present we are unable to predict them since they reflect long-distance dynamics.

- Observation of such an asymmetry (or lack thereof) will not provide us with reliable numerical information on the parameters of the microscopic theory, like the KM ansatz.

- Nevertheless comprehensive and detailed studies are an absolute must!

Later I will describe examples where the relevant long-distance parameters – phase shifts etc. – can be *measured* independantly.

CP Violation Involving Oscillations

The essential feature that a final state in this category has to satisfy is that it can be fed both by B^0 and $\bar{B}^0$ decays [n]. However for convenience reasons I will concentrate on a special subclass of such modes, namely when the final state is a CP eigenstate. A more comprehensive discussion can be found in [29,6].

Three qualitative observations have to be made here:

- Since the final state is shared by B^0 and $\bar{B}^0$ decays one cannot even define a CP asymmetry unless one acquires *independant* information on the decaying meson: was it a B^0 or $\bar{B}^0$ or – more to the point – was it originally produced as a B^0 or $\bar{B}^0$? There are several scenarios for achieving such *flavour tagging*:

  – Nature could do the trick for us by providing us with B^0 - $\bar{B}^0$ production asymmetries through, say, associated production in hadronic collisions or the use of polarized beams in e^+e^- annihilation. Those production asymmetries could be tracked through decays that are necessarily CP conserving – like $\bar{B}_d \to \psi K^- \pi^+$ vs. $B_d \to \psi K^+ \pi^-$. It seems unlikely, though, that such a scenario could ever be realized with sufficient statistics.

  – *Same Side Tagging*: One undertakes to repeat the success of the D^* tag for charm mesons – $D^{+*} \to D^0 \pi^+$ vs. $D^{-*} \to \bar{D}^0 \pi^-$ – through finding a conveniently placed nearby resonance – $B^{-**} \to \bar{B}_d \pi^-$ vs. $B^{+**} \to B_d \pi^+$ – or through employing correlations between the beauty mesons and a 'nearby' pion (or kaon for B_s) as pioneered by the CDF collaboration. This method can be calibrated by analysing how well $B^0 - \bar{B}^0$ oscillations are reproduced.

  – *Opposite Side Tagging*: With electromagnetic and strong forces conserving the beauty quantum number, one can employ charge correlations between the decay products (leptons and kaons) of the two beauty hadrons originally produced together.

  – If the lifetimes of the two mass eigenstates of the neutral B meson differ sufficiently from each other, then one can wait for the short-lived component to fade away relative to the long-lived one and proceed in qualitative analogy to the K_L case. Conceivably this could become feasible – or even essential – for overly fast oscillating B_s mesons [30].

  The degree to which this flavour tagging can be achieved is a crucial challenge each experiment has to face.

- The CP asymmetry is largest when the two interfering amplitudes are comparable in magnitude. With oscillations having to provide the second amplitude that is absent initially at time of production, the CP asymmetry starts out at zero for decays that occur right after production and builds up for later

[n] Obviously no such common channels can exist for charged mesons or for baryons.

decays. The (first) maximum of the asymmetry

$$\left|1-\frac{1-\mathrm{Im}\frac{q}{p}\bar{\rho}_f\sin\Delta m_B t}{1+\mathrm{Im}\frac{q}{p}\bar{\rho}_f\sin\Delta m_B t}\right| \tag{212}$$

is reached for

$$\frac{t}{\tau_B}=\frac{\pi}{2}\frac{\Gamma_B}{\Delta m_B}\simeq 2 \tag{213}$$

in the case of B_d mesons.

- The other side of the coin is that very rapid oscillations – $\Delta m_B \gg \Gamma_B$ as is the case for B_s mesons – will tend to wash out the asymmetry or at least will severely tax the experimental resolution.

On the Sign of CP Asymmetries involving Oscillations

Let us consider the asymmetry derived from Eq.(201) for decays into a final state f that is a CP eigenstate:

$$A_f \equiv \frac{\Gamma(\bar{B}_d(t)\to f)-\Gamma(B_d(t)\to f)}{\Gamma(\bar{B}_d(t)\to f)+\Gamma(B_d(t)\to f)}=(\sin\Delta m_B t)\cdot \mathrm{Im}\frac{q}{p}\bar{\rho}_f \tag{214}$$

Obviously one can measure both the size and the sign of A_f. Yet at first sight it would appear that no useful information can be extracted from the observed sign since it depends on Δm_B and the sign of the latter cannot be defined nor determined *experimentally* in a feasible way. For the two mass eigenstates in the $B_d/\bar{B}_d$ complex can be distinguished neither by an observable difference in lifetimes nor by their CP parities – unlike for kaons: (i) One confidently predicts $\Delta\Gamma_{B_d}/\Gamma_{B_d}$ to not exceed the percent level. Such a small difference cannot be observed in the foreseeable future. Remember that $\Gamma_{K_S}\gg\Gamma_{K_L}$ represents a kinematical accident. (ii) As emphasized before the KM ansatz predicts large CP violation in the $B_d/\bar{B}_d$ complex.

Yet the essential point is that within a given theory for $\Delta B=2$ dynamics one can nevertheless predict the overall sign of A_f!

Let us start from the general discussion in Sect.2.2. Using the conventions $q/p=+\sqrt{(M_{12}^*-\frac{i}{2}\Gamma_{12}^*)/(M_{12}-\frac{i}{2}\Gamma_{12})}$, $\mathbf{CP}|B_d\rangle=|\bar{B}_d\rangle$ one finds

$$\Delta m_{B_d}\simeq -2\bar{M}_{12}^{B_d} \tag{215}$$

like in the kaon case, albeit for a different reason, namely $|\bar{\Gamma}_{12}/\bar{M}_{12}|\ll 1$. One relies on the box diagram to derive $\mathcal{L}_{eff}(\Delta B=2)$ and evaluate Δm_{B_d} with it. This is similar to the procedure indicated above for Δm_K. There is actually better justification for Δm_B being produced by short-distance physics than Δm_K. Again we find

$$\Delta m_{B_d}\equiv m_B-m_A>0\ , \tag{216}$$

where the subscripts A and B label the two mass eigenstates. As stated above we are not able to characterize in an *empirical* way which of the two states is heavier. By

that I mean we can only say that one state is heavier than the other by an amount Δm_B; yet for practical limitations we cannot relate this finding to an *observable* difference in lifetimes or to *observable* CP parities. Next we consider the ratio of decay amplitude into a final state f that is a CP eigenstate: $\mathbf{CP}|f_\pm\rangle = \pm|f_\pm\rangle$:

$$\bar{\rho}_{f_\pm} = \frac{\langle f_\pm|H_{\Delta B=1}|\bar{B}_d\rangle}{\langle f_\pm|H_{\Delta B=-1}|B_d\rangle} \ , \quad \mathbf{CP} H_{\Delta B=1} (\mathbf{CP})^\dagger = H^*_{\Delta B=-1} \tag{217}$$

Since

$$\langle f_\pm|H_{\Delta B=1}|\bar{B}_d\rangle = \langle f_\pm|(\mathbf{CP})^\dagger \mathbf{CP} H_{\Delta B=1} (\mathbf{CP})^\dagger \mathbf{CP}|\bar{B}_d\rangle = \pm\langle f_\pm|H^*_{\Delta B=-1}|B_d\rangle \tag{218}$$

we find

$$\bar{\rho}_{f_\pm} = \pm \frac{\langle f_\pm|H^*_{\Delta B=-1}|B_d\rangle}{\langle f_\pm|H_{\Delta B=-1}|B_d\rangle} \tag{219}$$

where it is obviously essential to adopt the same convention $\mathbf{CP}|B_d\rangle = |\bar{B}_d\rangle$ used in evaluating Δm_{B_d}. Putting the pieces together we arrive at

$$A_{f_\pm} \simeq \pm\,(\sin|\Delta m_{B_d}|t) \cdot \mathrm{Im}\left(\sqrt{\frac{M^*_{12}}{M_{12}}} \frac{\langle f_\pm|H^*_{\Delta B=-1}|B_d\rangle}{\langle f_\pm|H_{\Delta B=-1}|B_d\rangle}\right) \tag{220}$$

where we have used $q/p \simeq \sqrt{\frac{M^*_{12}}{M_{12}}}$. We read off from Eq.(220) that knowing the CP parity of the final state $f_\pm$ we can deduce the sign of $\mathrm{Im}\frac{q}{p}\bar{\rho}_f$ from the *observed* sign of $A_{f_\pm}$. The ambiguity we have in the sign of Δm_B is thus compensated by a corresponding ambiguity in the sign of $\frac{q}{p}\bar{\rho}_f$.

This at first sight surprising result can be seen also in the following down-to-earth way:

- Going to a different phase convention by changing $q/p \to -q/p$ maintains the defining property $(q/p)^2 = (M^*_{12} - \frac{i}{2}\Gamma^*_{12})/(M_{12} - \frac{i}{2}\Gamma_{12})$, see Eq.(16). Observables thus cannot be affected.
- Yet the two mass eigenstates labeled by subscripts A and B exchange places, see Eq.(13).
- The difference $\Delta M = -2\mathrm{Re}\left[\frac{q}{p}(M_{12} - \frac{i}{2}\Gamma_{12})\right]$ then flips its sign – yet so does $\mathrm{Im}\frac{q}{p}\bar{\rho}_f$!
- The product $(\sin\Delta m_B t) \cdot \mathrm{Im}\frac{q}{p}\bar{\rho}_f$ therefore remains invariant.

Resume

Three classes of quantities each describe the three types of CP violation:

1.
$$\left|\frac{q}{p}\right| \neq 1 \tag{221}$$

2.
$$\left|\frac{T(\bar{B}\to\bar{f})}{T(B\to f)}\right| \neq 1 \tag{222}$$

3.
$$\mathrm{Im}\frac{q}{p}\frac{T(\bar{B}\to\bar{f})}{T(B\to f)} \neq 0 \tag{223}$$

These quantities obviously satisfy one necessary condition for being observables: they are insensitive to the phase convention adopted for the anti-state.

5.4 Parametric KM Predictions

The triangle defined by

$$\lambda V(cb) - V(ub) + V^*(td) = 0 \tag{224}$$

to leading order controls basic features of B transitions. As discussed before, it has naturally large angles; it usually is called *the* KM triangle. Its angles are given by KM matrix elements which are most concisely expressed in the Wolfenstein representation:

$$e^{i\phi_1} = -\frac{V(td)}{|V(td)|} \quad , \quad e^{i\phi_2} = \frac{V^*(td)}{|V(td)|}\frac{|V(ub)|}{V(ub)} \quad , \quad e^{i\phi_3} = \frac{V(ub)}{|V(ub)|} \tag{225}$$

The various CP asymmetries in beauty decays are expressed in terms of these three angles. I will describe 'typical' examples now.

Angle ϕ_1

Consider

$$\bar{B}_d \to \psi K_S \leftarrow B_d \tag{226}$$

where the final state is an almost pure odd CP eigenstate. On the quark level one has two different reactions, namely one describing the direct decay process

$$\bar{B}_d = [b\bar{d}] \to [c\bar{c}][s\bar{d}] \tag{227}$$

and the other one involving a $B_d - \bar{B}_d$ oscillation:

$$\bar{B}_d = [b\bar{d}] \Rightarrow B_d = [\bar{b}d] \to [c\bar{c}][\bar{s}d] \tag{228}$$

♠ ♠ ♠

Homework Problem # 5:

How can the $[s\bar{d}]$ combination in Eq.(227) interfere with $[\bar{s}d]$ in Eq.(228)?

♠ ♠ ♠

Since the final state in $B/\bar{B} \to \psi K_S$ can carry isospin 1/2 only, we have for the *direct* transition amplitudes:

$$\begin{aligned} T(\bar{B}_d \to \psi K_S) &= V(cb)V^*(cs)e^{i\alpha_{1/2}}|\mathcal{M}_{1/2}| \\ T(B_d \to \psi K_S) &= V^*(cb)V(cs)e^{i\alpha_{1/2}}|\mathcal{M}_{1/2}| \end{aligned} \tag{229}$$

and thus

$$\bar{\rho}_{\psi K_S} = \frac{V(cb)V^*(cs)}{V^*(cb)V(cs)} \tag{230}$$

from which the hadronic quantities, namely the phase shift $\alpha_{1/2}$ and the hadronic matrix element $|\mathcal{M}_{1/2}|$ – both of which *cannot* be calculated in a reliable manner – have dropped out. Therefore

$$|\bar{\rho}_{\psi K_S}| = \left|\frac{T(\bar{B}_d \to \psi K_S}{T(B_d \to \psi K_S}\right| = 1 \; ; \tag{231}$$

i.e., there can be *no direct* CP violation in this channel.

Since $|\Gamma_{12}| \ll |M_{12}|$ one has

$$\frac{q}{p} \simeq \sqrt{\frac{M_{12}^*}{M_{12}}} = \frac{M_{12}^*}{|M_{12}|} \simeq \frac{V^*(tb)V(td)}{V(tb)V^*(td)} \tag{232}$$

which is a pure phase. Altogether one obtains [o]

$$\mathrm{Im}\frac{q}{p}\bar{\rho}_{\psi K_S} = \mathrm{Im}\left(\frac{V^*(tb)V(td)}{V(tb)V^*(td)}\frac{V(cb)V^*(cs)}{V^*(cb)V(cs)}\right) \simeq \mathrm{Im}\frac{V^2(td)}{|V(td)|^2} = \sin 2\phi_1 \tag{233}$$

That means that to a very good approximation the observable Im $\frac{q}{p}\bar{\rho}_{\psi K_S}$, which is the amplitude of the oscillating CP asymmetry, is in general given by *microscopic* parameters of the theory; within the KM ansatz they combine to yield the angle ϕ_1 [27]. Within the Wolfenstein representation one has

$$\mathrm{Im}\frac{q}{p}\bar{\rho}_{\psi K_S} \simeq \frac{2\eta(1-\rho)}{(1-\rho)^2+\eta^2} > 0 \tag{234}$$

since the analysis of $K_L \to \pi\pi$ yields $\eta > 0$, $|\rho| < 1$, see Sect. 2.8. The first pilot studies yield [31,32]:

$$\mathrm{Im}\left(\frac{q}{p}\bar{\rho}_{\psi K_S}\right) = \begin{cases} 3.2\,^{+1.8}_{-2.0} \pm 0.5 \; , & \text{OPAL Collaboration} \\ 1.8 \pm 1.1 \pm 0.3 \; , & \text{CDF Collaboration} \end{cases} \tag{235}$$

Several other channels are predicted to exhibit a CP asymmetry expressed by $\sin 2\phi_1$, like $B_d \to \psi K_L$ [p], $B_d \to D\bar{D}$ etc.

♠ ♠ ♠

Homework Problem # 6:

At first sight it would seem that Eq.(233 cannot be correct since the quantity $\frac{V^*(tb)V(td)}{V(tb)V^*(td)}\frac{V(cb)V^*(cs)}{V^*(cb)V(cs)}$ is not invariant under changes in the phase conventions adopted for the d and s quark fields whereas observables like $\sin 2\phi_1$ have to be. There is a spurious factor not listed explicitely that takes care of this problem. Explain what it is. (*Hint*: Remember Homework Problem # 5.)

♠ ♠ ♠

---

[o] The next-to-last (approximate) equality in Eq.(233) holds in the Wolfenstein representation, although the overall result is general.

[p] Keep in mind that $\mathrm{Im}\frac{q}{p}\bar{\rho}_{\psi K_L} = -\mathrm{Im}\frac{q}{p}\bar{\rho}_{\psi K_S}$ holds because K_L is mainly CP odd and K_S mainly CP even.

Angle ϕ_2

The situation is not quite as clean for the angle ϕ_2. The asymmetry in $\bar{B}_d \to \pi^+\pi^-$ vs. $B_d \to \pi^+\pi^-$ is certainly sensitive to ϕ_2, yet there are two complications:

- The final state is described by a superposition of *two* different isospin states, namely $I = 0$ and 2. The spectator process contributes to both of them.
- The Cabibbo suppressed Penguin operator

$$b \to dg^* \to du\bar{u} \tag{236}$$

  will also contribute, albeit only to the $I = 0$ amplitude.

The direct transition amplitudes are then expressed as follows:

$$T(\bar{B}_d \to \pi^+\pi^-) = V(ub)V^*(ud)e^{i\alpha_2}|\mathcal{M}_2^{spect}|+$$

$$+ e^{i\alpha_0}\left(V(ub)V^*(ud)|\mathcal{M}_0^{spect}| + V(tb)V^*(td)|\mathcal{M}_0^{Peng}|\right) \tag{237}$$

$$T(B_d \to \pi^+\pi^-) = V^*(ub)V(ud)e^{i\alpha_2}|\mathcal{M}_2^{spect}|+$$

$$+ e^{i\alpha_0}\left(V^*(ub)V(ud)|\mathcal{M}_0^{spect}| + V^*(tb)V(td)|\mathcal{M}_0^{Peng}|\right) \tag{238}$$

where the phase shifts for the $I = 0, 2$ states have been factored off.

If there were no Penguin contributions, we would have

$$\mathrm{Im}\frac{q}{p}\bar{\rho}_{\pi\pi} = \mathrm{Im}\frac{V(td)V^*(tb)V(ub)V^*(ud)}{V^*(td)V(tb)V^*(ub)V(ud)} = -\mathrm{sin}2\phi_2 \tag{239}$$

without direct CP violation – $|\bar{\rho}_{\pi\pi}| = 1$ – since the two isospin amplitudes still contain the same weak parameters. The Penguin contribution changes the picture in two basic ways:

1. The CP asymmetry no longer depends on ϕ_2 alone:

$$\mathrm{Im}\frac{q}{p}\bar{\rho}_{\pi\pi} \simeq -\mathrm{sin}2\phi_2 + \left|\frac{V(td)}{V(ub)}\right| \left[\mathrm{Im}\left(e^{-i\phi_2}\frac{\mathcal{M}^{Peng}}{\mathcal{M}^{spect}}\right) - \mathrm{Im}\left(e^{-3i\phi_2}\frac{\mathcal{M}^{Peng}}{\mathcal{M}^{spect}}\right)\right] +$$

$$+ \mathcal{O}(|\mathcal{M}^{Peng}|^2/|\mathcal{M}^{spect}|^2) \tag{240}$$

   where

$$\mathcal{M}^{spect} = e^{i\alpha_0}|\mathcal{M}_0^{spect}| + e^{i\alpha_2}|\mathcal{M}_2^{spect}| \;\; , \;\; \mathcal{M}^{Peng} = e^{i\alpha_0}|\mathcal{M}_0^{Peng}| \tag{241}$$

2. A direct CP asymmetry emerges:

$$|\bar{\rho}_{\pi\pi}| \neq 1 \tag{242}$$

Since we are dealing with a Cabibbo suppressed Penguin operator, we expect that its contribution is reduced relative to the spectator term:

$$\left| \frac{\mathcal{M}^{Peng}}{\mathcal{M}^{spect}} \right| < 1 \;, \tag{243}$$

which was already used in Eq.(240). Unfortunately this reduction might not be very large. This concern is based on the observation that the branching ratio for $\bar{B}_d \to K^- \pi^+$ appears to be somewhat larger than for $\bar{B}_d \to \pi^+ \pi^-$ implying that the Cabibbo favoured Penguin amplitude is at least not smaller than the spectator amplitude.

Various strategies have been suggested to unfold the Penguin contribution through a combination of additional or other measurements (of other $B \to \pi\pi$ channels or of $B \to \pi\rho$, $B \to K\pi$ etc.) and supplemented by theoretical considerations like $SU(3)_{Fl}$ symmetry [33]. I am actually hopeful that the multitude of exclusive nonleptonic decays (which is the other side of the coin of small branching ratios!) can be harnessed to extract a wealth of information on the strong dynamics that in turn will enable us to extract sin$2\phi_2$ with decent accuracy.

The ϕ_3 Saga

Of course it is important to determine ϕ_3 as accurately as possible. This will not be easy, and one better keep a proper perspective. I am going to tell this saga now in two installments.

(I) *CP asymmetries involving $B_s - \bar{B}_s$ Oscillations*: In principle one can extract ϕ_3 from KM suppressed B_s decays like one does ϕ_2 from B_d decays, namely by measuring and analyzing the difference between the rates for, say, $\bar{B}_s(t) \to K_S \rho^0$ and $B_s(t) \to K_S \rho^0$: Im$\frac{q}{p}\bar{\rho}_{K_S\rho^0} \sim$ sin$2\phi_3$. One has to face the same complication, namely that in addition to the spectator term a (Cabibbo suppressed) Penguin amplitude contributes to $\bar{\rho}_{K_S\rho^0}$ with different weak parameters. Yet the situation is much more challenging due to the rapid pace of the $B_s - \bar{B}_s$ oscillations.
A more promising way might be to compare the rates for $\bar{B}_s(t) \to D_s^+ K^-$ with $B_s(t) \to D_s^- K^+$ as a function of the time of decay t since there is no Penguin contribution. The asymmetry depends on sinϕ_3 rather than sin$2\phi_3$ [q]

(II) *Direct CP Asymmetries*: The largish direct CP asymmetries sketched above for $B \to K\pi$ depend on sinϕ_3 – and on the phase shifts which in general are neither known nor calculable. Yet in some cases they can be determined experimentally – as first described for $B^\pm \to D_{neutral} K^\pm$ [34]. There are *four independant* rates that can be measured, namely

$$\Gamma(B^- \to D^0 K^-) \;,\; \Gamma(B^- \to \bar{D}^0 K^-) \;,\; \Gamma(B^- \to D_\pm K^-) \;,\; \Gamma(B^+ \to D_\pm K^+) \tag{244}$$

The *flavour eigenstates* D^0 and $\bar{D}^0$ are defined through flavour specific modes, namely $D^0 \to l^+ X$ and $\bar{D}^0 \to l^- X$, respectively; $D_\pm$ denote the even/odd CP

[q] Both $D_s^+ K^-$ and $D_s^- K^+$ are final states common to B_s and $\bar{B}_s$ decays although they are not CP eigenstates.

eigenstates $D_\pm = (D^0 \pm \bar{D}^0)/\sqrt{2}$ defined by $D_+ \to K^+K^-$, $\pi^+\pi^-$, etc., $D_- \to K_S\pi^0$, $K_S\eta$, etc. [35].

From these four observables one can (up to a binary ambiguity) extract the four basic quantities, namely the moduli of the two independant amplitudes ($|T(B^- \to D^0K^-)|$, $|T(B^- \to \bar{D}^0K^-)|$), their strong phaseshift – and sinϕ_3, the goal of the enterprise!

A Zero-Background Search for New Physics: $B_s \to \psi\phi$, $D_s^+D_s^-$

The two angles ϕ_1 and ϕ_2 will be measured in the next several years with decent or even good accuracy. I find it unlikely that any of the direct measurements of ϕ_3 sketched above will yield a more precise value than inferred from simple trigonometry:

$$\phi_3 = 180^o - \phi_1 - \phi_2 \tag{245}$$

Eq.(245) holds within the KM ansatz; of course the real goal is to uncover the intervention of New Physics in B_s transitions. It then makes eminent sense to search for it in a reaction where Known Physics predicts a practically zero result. $B_s \to \psi\phi$, $\psi\eta$, $D_s\bar{D}_s$ fit this bill [27]: to leading order in the KM parameters the CP asymmetry has to vanish since on that level quarks of the second and third family only participate in $B_s - \bar{B}_s$ oscillations – $[s\bar{b}] \Rightarrow t^*\bar{t}^* \Rightarrow [b\bar{s}]$ – and in these direct decays – $[b\bar{s}] \to c\bar{c}s\bar{s}$. Any CP asymmetry is therefore Cabibbo suppressed, i.e. $\leq 4\%$. More specifically

$$\mathrm{Im}\frac{q}{p}\bar{\rho}_{B_s\to\psi\eta,\psi\phi,D_s\bar{D}_s}\Big|_{KM} \sim 2\% \tag{246}$$

Yet New Physics has a good chance to contribute to $B_s - \bar{B}_s$ oscillations; if so, there is no reason for it to conserve CP and asymmetries can emerge that are easily well in excess of 2% . New Physics scenarios with non-minimal SUSY or flavour-changing neutral currents could actually yield asymmetries of $\sim 10 \div 30\%$ [36] – completely beyond the KM reach!

A Menu for Gourmets

Quite often people in the US tend to believe that a restaurant that presents them with a long menu must be a very good one. The real experts – like the French and Italians – of course know better: it is the hallmark of a top cuisine to concentrate on a few very special dishes and prepare them in a spectacular fashion rather than spread one's capabilities too thinly.There is a first class menu consisting of three main dishes and one side dish, namely

1. measure $\Delta m(B_s)$ which within the Standard Model allows to extract $|V(td)|$ through
$$\frac{\Delta m(B_d)}{\Delta m(B_s)} \simeq \frac{Bf^2_{B_d}}{Bf^2_{B_s}}\left|\frac{V(td)}{V(ts)}\right|^2 \; ; \tag{247}$$
2. determine the rates for $\bar{B}_d \to \psi K_S$ and $B_d \to \psi K_S$ to obtain the value of sin$2\phi_1$;

3. compare $\bar{B}_s \to \psi\phi,\ D_s\bar{D}_s$ with $B_s \to \psi\phi,\ D_s\bar{D}_s$ as a clean search for New Physics and

4. as a side dish: measure the B_s lifetime separately in $B_s \to l\nu D_s^{(*)}$ and $B_s \to \psi\phi,\ D_s\bar{D}_s$ where the former yields the algebraic average of the $B_{s,short}$ and $B_{s,long}$ lifetimes and the latter the $B_{s,short}$ lifetime. One predicts for them [37]:

$$\frac{\tau(B_s \to l\nu D_s^{(*)}) - \tau(B_s \to \psi\phi,\ D_s\bar{D}_s)}{\tau(B_s \to l\nu D_s^{(*)})} \simeq 0.1 \cdot \left(\frac{f_{B_s}}{200\ \mathrm{MeV}}\right)^2 \tag{248}$$

This menu featuring B_s decays so prominently can be prepared at hadronic machines only. Thus it represents a task for HERA-B, CDF/D0 and later LHCB and BTeV.

While the experiments have to exhibit the voracious appetite of a gourmand to gobble up enough statistics, they have to demonstrate the highly discriminating taste of a gourmet to succeed!

5.5 KM Trigonometry

One side of the triangle is exactly known since the base line can be normalized to unity without affecting the angles:

$$1 - \frac{V(ub)}{\lambda V(cb)} + \frac{V^*(td)}{\lambda V(cb)} = 0 \tag{249}$$

The second side is known to some degree from semileptonic B decays:

$$\left|\frac{V(ub)}{V(cb)}\right| \simeq 0.08 \pm 0.03 \tag{250}$$

where the quoted uncertainty is mainly theoretical and amounts to little more than a guestimate. In the Wolfenstein representation this reads as

$$\sqrt{\rho^2 + \eta^2} \simeq 0.38 \pm 0.11 \tag{251}$$

The area cannot vanish since $\epsilon_K \neq 0$. Yet at present not much more can be said for certain.

In principle one would have enough observables – namely ϵ_K and $\Delta m(B_d)$ in addition to $|V(ub)/V(cb)|$ – to determine the two KM parameters ρ and η in a *redundant* way. In practise, though, there are two further unknowns, namely the size of the $\Delta S = 2$ and $\Delta B = 2$ matrix elements, as expressed through B_K and $B_B f_B^2$. For m_t sufficiently large ϵ_K is dominated by the top contribution: $d\bar{s} \Rightarrow t^*\bar{t}^* \Rightarrow s\bar{d}$. The same holds always for $\Delta m(B_d)$. In that case things are simpler:

$$\frac{|\epsilon_K|}{\Delta m(B_d)} \propto \sin 2\phi_1 \simeq 0.42 \cdot UNC \tag{252}$$

with the factor UNC parametrising the uncertainties

$$UNC \simeq \left(\frac{0.04}{|V(cb)|}\right)\left(\frac{0.72}{x_d}\right) \cdot \left(\frac{\eta_{QCD}^{(B)}}{0.55}\right) \cdot \left(\frac{0.62}{\eta_{QCD}^{(K)}}\right) \cdot \left(\frac{2B_B}{3B_K}\right) \cdot \left(\frac{f_B}{160\,\mathrm{MeV}}\right)^2 \tag{253}$$

where $x_d \equiv \Delta m(B_d)/\Gamma(B_d)$; $\eta^{(B)}_{QCD}$ and $\eta^{(K)}_{QCD}$ denote the QCD radiative corrections for $\mathcal{H}(\Delta B = 2)$ and $\mathcal{H}(\Delta S = 2)$, respectively; B_B and B_K express the expectation value of $\mathcal{H}(\Delta B = 2)$ or $\mathcal{H}(\Delta S = 2)$ in units of the 'vacuum saturation' result which is given in terms of the decay constants f_B and f_K (where the latter is known). The main uncertainty is obviously of a theoretical nature related to the hadronic parameters B_B, B_K and f_B; as discussed before, state-of-the-art theoretical technologies yield $B_B \simeq 1$, $B_K \simeq 0.8 \pm 0.2$ and $f_B \simeq 180 \pm 30\,\mathrm{MeV}$ where the latter range might turn out to be anything but conservative! Eq.(252) represents an explicit illustration that some CP asymmetries in B^0 decays are huge.

For $m_t \simeq 180$ GeV the $c\bar{c}$ and $c\bar{t} + t\bar{c}$ contributions to ϵ_K are still sizeable; nevertheless Eq.(252) provides a good approximation. Furthermore sin$2\phi_1$ can still be expressed reliably as a function of the hadronic matrix elements:

$$\sin 2\phi_1 = f(B_B f_B^2 / B_K) \tag{254}$$

It will become obvious why this is relevant.

The general idea is, of course, to construct the triangle as accurately as possible and then probe it; i.e. search for inconsistencies that would signal the intervention of New Physics. A few remarks on that will have to suffice here.

As indicated before we can expect the value of $|V(ub)/V(cb)|$ to be known to better than 10% and hope for $|V(td)|$ to be determined with decent accuracy as well. The triangle will then be well determined or even overdetermined. Once the first asymmetry in B decays that can be interpreted reliably – say in $B_d \to \psi K_S$ – has been measured and ϕ_1 been determined, the triangle is fully constructed from B decays alone. Furthermore one has arrived at the first sensitive consistency check of the triangle: one compares the measured value of sin$2\phi_1$ with Eq.(252) to infer which value of $B_B f_B^2$ is thus required; this value is inserted into the Standard Model expression for $\Delta m(B_d)$ together with m_t to see whether the experimental result is reproduced.

A host of other tests can be performed that are highly sensitive to

- the presence of New Physics and
- to some of their salient dynamical features.

Details can be found in the ample literature on that subject.

6 Oscillations and CP Violation in Charm Decays – The Underdog's Chance for Fame

It is certainly true that

- $D^0 - \bar{D}^0$ oscillations proceed very slowly in the Standard Model and
- CP asymmetries in D decays are small or even tiny within the KM ansatz.

Yet the relevant question quantitatively is: how slow and how small?

6.1 $D^0 - \bar{D}^0$ Oscillations

Bounds on $D^0 - \bar{D}^0$ oscillations are most cleanly expressed through 'wrong-sign' semileptonic decays:

$$r_D = \frac{\Gamma(D^0 \to l^- X)}{\Gamma(D^0 \to l^+ X)} \simeq \frac{1}{2}\left(x_D^2 + y_D^2\right) \tag{255}$$

with $x_D = \Delta m_D/\Gamma_D$, $y_D = \Delta\Gamma_D/2\Gamma_D$. It is often stated that the Standard Model predicts

$$r_D \leq 10^{-7} \;\hat{=}\; x_D, \, y_D \leq 3 \cdot 10^{-4} \tag{256}$$

I myself am somewhat flabbergasted by the boldness of such predictions. For one should keep the following in mind for proper perspective: there are quite a few channels that can drive $D^0 - \bar{D}^0$ oscillations – like $D^0 \Rightarrow K\bar{K}$, $\pi\pi \Rightarrow \bar{D}^0$ or $D^0 \Rightarrow K^-\pi^+ \Rightarrow \bar{D}^0$ – and they branching ratios on the (few) $\times\, 10^{-3}$ level.[r] In the limit of $SU(3)_{Fl}$ symmetry all these contributions have to cancel of course. Yet there are sizeable violations of $SU(3)_{Fl}$ invariance in D decays, and one should have little confidence in an imperfect symmetry to ensure that a host of channels with branching ratios of order few$\times 10^{-3}$ will cancel as to render x_D, $y_D \leq 3 \cdot 10^{-4}$. To say it differently: The relevant question in this context is *not* whether $r_D \sim 10^{-7} \div 10^{-6}$ is a possible or even reasonable Standard Model estimate, but whether $10^{-6} \leq r_D \leq 10^{-4}$ can *reliably be ruled out*! I cannot see how anyone could make such a claim with the required confidence.

The present experimental bound is

$$r_D|_{exp} \leq 3.4 \cdot 10^{-3} \;\hat{=}\; x_D, \, y_D \leq 0.1 \tag{257}$$

to be compared with a *conservative* Standard Model bound

$$r_D|_{SM} < 10^{-4} \;\hat{=}\; y_D, \, x_D|_{SM} \leq 10^{-2} \tag{258}$$

New Physics on the other hand can enhance Δm_D (though not $\Delta\Gamma_D$) very considerably up to

$$x_D|_{NP} \sim 0.1 \; , \tag{259}$$

i.e. the present experimental bound.

6.2 CP Violation involving $D^0 - \bar{D}^0$ Oscillations

One can discuss this topic in close qualitative analogy to B decays. First one considers final states that are CP eigenstates like K^+K^- or $\pi^+\pi^-$ [40]:

$$\text{rate}(D^0(t) \to K^+K^-) \propto e^{-\Gamma_D t}\left(1 + (\sin\Delta m_D t) \cdot \text{Im}\frac{q}{p}\bar{\rho}_{K^+K^-}\right) \simeq$$

$$\simeq e^{-\Gamma_D t}\left(1 + \frac{\Delta m_D t}{\Gamma_D} \cdot \frac{t}{\tau_D} \cdot \text{Im}\frac{q}{p}\bar{\rho}_{K^+K^-}\right) \tag{260}$$

[r] For the $K^-\pi^+$ mode this represents the average of its Cabibbo allowed and doubly Cabibbo suppressed incarnations.

With $x_D|_{SM} \leq 10^{-2}$ and $\mathrm{Im}\frac{q}{p}\bar{\rho}_{K^+K^-}|_{KM} \sim \mathcal{O}(10^{-3})$ one arrives at an asymmetry of around 10^{-5}, i.e. for all practical purposes zero, since it presents the product of two very small numbers. Yet with New Physics one conceivably has $x_D|_{NP} \leq 0.1$, $\mathrm{Im}\frac{q}{p}\bar{\rho}_{K^+K^-}|_{NP} \sim \mathcal{O}(10^{-1})$ leading to an asymmetry that could be as large as of order 1%. Likewise one should compare the doubly Cabibbo suppressed transitions [41,42]

$$\mathrm{rate}(D^0(t) \to K^+\pi^-) \propto e^{-\Gamma_{D^0}t}\mathrm{tg}^4\theta_C|\hat{\rho}_{K\pi}|^2 \cdot$$
$$\cdot\left[1 - \frac{1}{2}\Delta\Gamma_D t + \frac{(\Delta m_D t)^2}{4\mathrm{tg}^4\theta_C|\hat{\rho}_{K\pi}|^2} + \frac{\Delta\Gamma_D t}{2\mathrm{tg}^2\theta_C|\hat{\rho}_{K\pi}|}\mathrm{Re}\left(\frac{p}{q}\frac{\hat{\rho}_{K\pi}}{|\hat{\rho}_{K\pi}|}\right) - \right.$$
$$\left. - \frac{\Delta m_D t}{\mathrm{tg}^2\theta_C|\hat{\rho}_{K\pi}|}\mathrm{Im}\left(\frac{p}{q}\frac{\hat{\rho}_{K\pi}}{|\hat{\rho}_{K\pi}|}\right)\right] \tag{261}$$
$$\mathrm{rate}(\bar{D}^0(t) \to K^-\pi^+) \propto e^{-\Gamma_{D^0}t}\mathrm{tg}^4\theta_C|\hat{\bar{\rho}}_{K\pi}|^2 \cdot$$
$$\cdot\left[1 - \frac{1}{2}\Delta\Gamma_D t + \frac{(\Delta m_D t)^2}{4\mathrm{tg}^4\theta_C|\hat{\bar{\rho}}_{K\pi}|^2} + \frac{\Delta\Gamma_D t}{2\mathrm{tg}^2\theta_C|\hat{\bar{\rho}}_{K\pi}|}\mathrm{Re}\left(\frac{p}{q}\frac{\hat{\bar{\rho}}_{K\pi}}{|\hat{\bar{\rho}}_{K\pi}|}\right) + \right.$$
$$\left. + \frac{\Delta m_D t}{\mathrm{tg}^2\theta_C|\hat{\bar{\rho}}_{K\pi}|}\mathrm{Im}\left(\frac{p}{q}\frac{\hat{\bar{\rho}}_{K\pi}}{|\hat{\bar{\rho}}_{K\pi}|}\right)\right] \tag{262}$$

where

$$\mathrm{tg}^2\theta_C \cdot \hat{\rho}_{K\pi} \equiv \frac{T(D^0 \to K^+\pi^-)}{T(D^0 \to K^-\pi^+)} \ , \ \ \mathrm{tg}^2\theta_C \cdot \hat{\bar{\rho}}_{K\pi} \equiv \frac{T(\bar{D}^0 \to K^-\pi^+)}{T(\bar{D}^0 \to K^+\pi^-)} \ ; \tag{263}$$

in such New Physics scenarios one would expect a considerably enhanced asymmetry of order $1\%/\mathrm{tg}^2\theta_C \sim 20\%$ – at the cost of smaller statistics.

Effects of that size would unequivocally signal the intervention of New Physics!

6.3 Direct CP Violation

As explained before a direct CP asymmetry requires the presence of two coherent amplitudes with different weak and different strong phases. Within the Standard Model (and the KM ansatz) such effects can occur in Cabibbo suppressed [s]; yet not in Cabibbo allowed or doubly Cabibbo suppressed modes. There is a subtlety involved in this statement. Consider for example $D^+ \to K_S\pi^+$. At first sight it appears to be a Cabibbo allowed mode described by a single amplitude without the possibility of an asymmetry. However [43]

- due to $K^0 - \bar{K}^0$ mixing the final state can be reached also through a doubly Cabibbo suppressed reaction, and the two amplitudes necessarily interfere;
- because of the CP violation in the $K^0 - \bar{K}^0$ complex there is an asymmetry that can be predicted on general grounds

$$\frac{\Gamma(D^+ \to K_S\pi^+) - \Gamma(D^- \to K_S\pi^-)}{\Gamma(D^+ \to K_S\pi^+) + \Gamma(D^- \to K_S\pi^-)} \simeq -2\mathrm{Re}\,\epsilon_K \simeq -3.3 \cdot 10^{-3} \simeq$$

[s]The effect could well reach the 10^{-3} and exceptionally the 10^{-2} level.

$$\simeq \frac{\Gamma(D^+ \to K_L\pi^+) - \Gamma(D^- \to K_L\pi^-)}{\Gamma(D^+ \to K_L\pi^+) + \Gamma(D^- \to K_L\pi^-)} \; ; \tag{264}$$

- If New Physics contributes to the doubly Cabibbo suppressed amplitude $D^+ \to K^0\pi^+$ (or $D^- \to \bar{K}^0\pi^-$) then an asymmetry could occur quite conceivably on the few percent scale;

- such a manifestation of New Physics would be unequivocal; against the impact of ϵ_K, Eq.(264) it could be distinguished not only through the size of the asymmetry, but also how it surfaces in $D^+ \to K_L\pi^+$ vs. $D^- \to K_L\pi^-$: if it is New Physics one has

$$\frac{\Gamma(D^+ \to K_S\pi^+) - \Gamma(D^- \to K_S\pi^-)}{\Gamma(D^+ \to K_S\pi^+) + \Gamma(D^- \to K_S\pi^-)} =$$

$$= -\frac{\Gamma(D^+ \to K_L\pi^+) - \Gamma(D^- \to K_L\pi^-)}{\Gamma(D^+ \to K_L\pi^+) + \Gamma(D^- \to K_L\pi^-)} \tag{265}$$

  i.e., the CP asymmetries in $D \to K_S\pi$ and $D \to K_L\pi$ differ in sign – in contrast to Eq.(264).

7 Heavy Quark Expansions (HQE)

7.1 Overview

One other intriguing and gratifying aspect of heavy flavour decays has become understood just over the last several years. It concerns primarily the strong interactions rather than the weak interactions: the decays in particular of beauty *hadrons* can be treated with a reliability and accuracy that before would have seemed unattainable. These new theoretical technologies can be referred to as *Heavy Quark Theory*; it combines two basic elements, namely an asymptotic symmetry principle and a dynamical treatment telling us how the asymptotic limit is approached:

- The symmetry principle is *Heavy Quark Symmetry* stating that all sufficiently heavy quarks behave identically under the strong interactions. Its origin can be understood in an intuitive way: consider a hadron H_Q containing a heavy quark Q with mass $m_Q \gg \Lambda_{QCD}$ surrounded by a "cloud" of light degrees of freedom carrying quantum numbers of an antiquark $\bar{q}$ or diquark qq [t]. This cloud has a rather complex structure: in addition to $\bar{q}$ (for mesons) or qq (for baryons) it contains an indefinite number of $q\bar{q}$ pairs and gluons that are strongly coupled to and constantly fluctuate into each other. There is, however, one thing we know: since typical frequencies of these fluctuations are $\sim \mathcal{O}(\text{few} \times \Lambda_{QCD}$, the normally dominant *soft* dynamics allow the heavy quark to exchange momenta of order few times Λ_{QCD} only with its surrounding medium. $Q\bar{Q}$ pairs then cannot play a significant role, and the heavy quark can be treated as a quantum mechanical object rather than a field theoretic

[t]This cloud is often referred to – somewhat disrespectfully – as 'brown muck'.

entity requiring second quantization. This provides a tremendous computational simplification even while maintaining a field theoretic description for the light degrees of freedom. Furthermore techniques developed long ago in QED can profitably be adapted here.

- We can go further and describe the interactions between Q and its surrounding light degrees of freedom through an expansion in powers of $1/m_Q$. This allows us to analyze *pre*-asymptotic effects, i.e. effects that fade away like a power of $1/m_Q$ as $m_Q \to \infty$.

This situation is qualitatively similar to chiral considerations which start from the limit of chiral invariance and describe the deviations from it through chiral perturbation theory. In both cases one has succeeded in describing nonperturbative dynamics in special cases.

The lessons we have learnt can be summarized as follows [38,39]: we have

- identified the sources of the non-perturbative corrections;
- found them to be smaller than they could have been;
- succeeded in relating the basic quantities of the Heavy Quark Theory – KM paramters, masses and kinetic energy of heavy quarks, etc. – to various a priori independant observables with a fair amount of redundancy;
- developed a better understanding of incorporating perturbative and nonperturbative corrections without double-counting.

In the following I will sketch the concepts on which the Heavy Quark Expansions are based, the techniques employed, the results obtained and the problems encountered. It will not constitute a self-sufficient introduction into this vast and ever expanding field. My intent is to provide a vademecum that

- creates an interest in the uninitiated for further reading and can serve as a guide for such a journey or
- refreshes the memory of readers who have heard it before while also pointing out the present frontline.

7.2 Theoretical Tools and Concepts

In describing weak decays of heavy flavour *hadrons* one has to incorporate perturbative as well as nonperturbative contributions in a self-consistent and complete way. The only known way to tackle such a task invokes the *Operator Product Expansion a la Wilson* involving an *effective* Lagrangian. Further conceptual insights as well as practical results can be gained by analysing *sum rules*; in particular they shed light on various aspects and formulations of *quark-hadron duality.*

Effective Lagrangians

The Standard Model defines the interaction driving beauty decays at an ultraviolet scale at or above the W boson mass M_W in a world with quarks, gluons and weak bosons. Yet in describing the decays of *hadrons*, we have to evaluate matrix elements of transition operators at ordinary hadronic scales. Those are much lower, namely around 1 GeV or so, at which point the weak bosons and heavy quarks have long ceased to represent dynamical entities. Such changes arise naturally through *effective* Lagrangians. In a quantum field theory one has to define a (renormalizable) Lagrangian in terms of certain fields at an ultraviolet scale M_{UV}. When considering this Lagrangian at a lower energy scale μ, all modes with characteristic frequencies *above* μ have to be 'integrated out' leaving only modes with *lower* frequencies as quantum *fields*. Yet the heavy degrees of freedom leave their mark in two ways:

- They induce new nonrenormalizable couplings among the light fields which scale like $(\mu/M_{UV})^{d-4}$ with d denoting the dimension of the new interaction operator. In particular removing the W boson fields gives rise to dimension six current-current operators.
- They affect the coefficients of the operators in the emerging Lagrangian. For example integrating out t, b, ... quark fields generates an imaginary part in the coefficients of the $\Delta B = 1, 2$ (and $\Delta S = 1, 2$) operators.

The scale μ separates short and long distance dynamics

$$\text{short distance} \;\; < \;\; \mu^{-1} \;\; < \;\; \text{long distance} \tag{266}$$

with the former entering through the coefficients and the latter through the effective operators; their matrix elements will thus depend on μ.

In principle the value of μ does not matter: it reflects merely our computational procedure rather than how nature goes about its business. The μ dependance of the coefficients thus has to cancel against that of the corresponding matrix elements.

In practise however there are competing demands on the choice of μ:

- On one hand one has to choose

  $$\mu \gg \Lambda_{QCD} \; ; \tag{267}$$

  otherwise radiative corrections cannot be treated within *perturbative* QCD.
- On the other hand many computational techniques for evaluating *matrix elements* – among them the Heavy Quark Expansions – require

  $$\mu \ll m_b \tag{268}$$

All of this has been well known for a long time, of course – in principle. Yet various subtleties that usually had been ignored became quite relevant when deriving effective Lagrangians for QCD itself with heavy quarks:

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^{light} + \mathcal{L}^{heavy} \tag{269}$$

$$\mathcal{L}_{QCD}^{light} = -\frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a + \sum_q \bar{q} i \not{D} q \ \ , \ \mathcal{L}^{heavy} = \sum_Q \bar{Q}(i \not{D} - m_Q)Q \ . \tag{270}$$

$G_{\mu\nu}^a$ denotes the gluon field-strength tensor, D_μ the covariant derivative, q the light quark fields u, d and s (for simplicity assumed to be massless) and Q the heavy quark fields, certainly b and possibly c. The heavy quark Lagrangian can be expanded in powers of $1/m_Q$:

$$\mathcal{L}^{heavy} = \sum_Q \left[\bar{Q}(i \not{D} - m_Q)Q + \frac{c_G}{2m_Q}\bar{Q}\frac{i}{2}\sigma \cdot GQ + \sum_{q,\Gamma} \frac{d_{Qq}^{(\Gamma)}}{m_Q^2}\bar{Q}\Gamma Q \bar{q}\Gamma q \right] + \mathcal{O}\left(1/m_Q^3\right) \tag{271}$$

where c_G and $d_{Qq}^{(\Gamma)}$ are coefficient functions, the Γ denote the possible Lorentz covariant fermion bilinears and $\sigma \cdot G = \sigma_{\mu\nu} G_{\mu\nu}$ with $G_{\mu\nu} = g t^a G_{\mu\nu}^a$. Thus a dimension five operator arises – usually referred to as *chromomagnetic* operator – and various dimension six four-fermion operators. In expanding expectation values later we will encounter also the so-called kinetic energy operator of dimension five

$$O_{kin} = \bar{Q}\vec{\pi}^2 Q \ \ , \ \vec{\pi} = -i\vec{D} \ ; \tag{272}$$

since it is not Lorentz invariant, it cannot appear in the Lagrangian.

Operator Product Expansion (OPE) for Inclusive Weak Decays

Similar to the well-known case of $\sigma(e^+e^- \to had)$ one invokes the optical theorem to describe the decay into a sufficiently *inclusive* final state f through the imaginary part of the forward scattering operator evaluated to second order in the weak interactions

$$\hat{T}(Q \to Q) = \mathrm{Im} \int d^4x \, i\{\mathcal{L}_W(x)\mathcal{L}_W^\dagger(0)\}_T \tag{273}$$

with the subscript T denoting the time-ordered product and $\mathcal{L}_W$ the relevant weak Lagrangian.[u] The expression in Eq.(273) represents in general a non-local operator with the space-time separation x being fixed by the inverse of the energy release. If the latter is large compared to typical hadronic scales, then the product is dominated by short-distance physics, and one can apply an operator product expansion a la Wilson on it yielding an infinite series of *local* operators of increasing dimension.[v] The width for the decay of a hadron H_Q containing Q is then obtained by taking the H_Q expectation value of the operator $\hat{T}$:

$$\frac{\langle H_Q | \mathrm{Im}\hat{T}(Q \to f \to Q) | H_Q \rangle}{2M_{H_Q}} \propto \Gamma(H_Q \to f) = \frac{G_F^2 m_Q^5}{192\pi^3}|V_{CKM}|^2 \cdot$$

[u] There are two qualitative differences to the case of $e^+e^- \to had$: in describing weak decays of a hadron H_Q (i) one employs the weak rather than the electromagnetic Lagrangian, and (ii) one takes the expectation value between the H_Q state rather than the vacuum.

[v] I will formulate the expansion in powers of $1/m_Q$, although it has to be kept in mind that it is really controlled by the inverse of the energy release. While there is no fundamental difference between the two for $b \to c/ul\bar{\nu}$ or $b \to c/u\bar{u}d$, since m_b, $m_b - m_{c,u} \gg \Lambda_{QCD}$, the expansion becomes of somewhat dubious reliability for $b \to c\bar{c}s$. It actually would break down for a scenario $Q_2 \to Q_1 l\bar{\nu}$ with $m_{Q_2} \simeq m_{Q_1}$ – in contrast to HQET!

$$\cdot \left[c_3^{(f)}(\mu) \frac{\langle H_Q|\bar{Q}Q|H_Q\rangle_{(\mu)}}{2M_{H_Q}} + \frac{c_5^{(f)}(\mu)}{m_Q^2} \frac{\langle H_Q|\bar{Q}\frac{i}{2}\sigma\cdot GQ|H_Q\rangle_{(\mu)}}{2M_{H_Q}} + \right.$$

$$\left. + \sum_i \frac{c_{6,i}^{(f)}(\mu)}{m_Q^3} \cdot \frac{\langle H_Q|(\bar{Q}\Gamma_i q)(\bar{q}\Gamma_i Q)|H_Q\rangle_{(\mu)}}{2M_{H_Q}} + \mathcal{O}(1/m_Q^4) \right] \quad (274)$$

Eq.(274) exhibits the following important features:

- As stated before in general terms, *short-distance* dynamics shape the c number coefficients $c_i^{(f)}$. *In practise* they are evaluated in *perturbative* QCD. It is quite conceivable, though, that also *nonperturbative* contributions arise there; yet they are believed to be fairly small in beauty decays [44].

- Nonperturbative contributions on the other hand enter through the *expectation values* of operators of dimension higher than three – $\bar{Q}\frac{i}{2}\sigma\cdot GQ$ etc. – and higher order corrections to the expectation value of the leading operator $\bar{Q}Q$, see next.

- I will describe later what we know about these expectation values. Let me just anticipate that $\langle H_Q|\bar{Q}Q|H_Q\rangle_{(\mu)}/2M_{H_Q} = 1+\mathcal{O}(1/m_Q^2)$. One then realize that the free quark model expression emerges asymptotically for the total width, i.e. for $m_Q \to \infty$.

- These nonperturbative contributions which are power suppressed can be described only if considerable care is applied in treating the *parametrically larger* perturbative corrections.

- The *leading* nonperturbative corrections arise at order $1/m_Q^2$ only. That means they are rather small in beauty decays since $(\mu/m_Q)^2 \sim$ few % for $\mu \leq 1$ GeV.

- Explicitely flavour dependant effects arise in order $1/m_Q^3$. They mainly drive the differences in the lifetimes of the various mesons of a given heavy flavour.

The absence of corrections of order $1/m_Q$ is particularly noteworthy and intriguing since such corrections do exist for hadronic masses – $M_{H_Q} = m_Q(1 + \bar{\Lambda}/m_Q + \mathcal{O}(1/m_Q^2))$ – and those control the phase space. Technically this follows from the fact that there is no *independant* dimension-four operator that could emerge in the OPE [w]. This result can be illuminated in more physical terms as follows. Bound-state effects in the initial state like mass shifts do generate corrections of order $1/m_Q$ to the total width; yet so does hadronization in the final state. *Local* colour symmetry demands that those effects cancel each other out. *It has to be emphasized that the absence of corrections linear in $1/m_Q$ is an unambiguous consequence of the OPE description. If* their presence were forced upon us, we would have encountered a *qualitative* change in our QCD paradigm. A discussion of this point has arisen recently phrased in the terminology of quark-hadron duality. I will return to this point later.

[w] The operator $\bar{Q}i\not{D}Q$ can be reduced to the leading operator $\bar{Q}Q$ through the equation of motion.

Sum Rules

Semileptonic decays of H_Q hadrons can be viewed as the crossed version of the deep-inelastic scattering of leptons off an H_Q target. Pursuing this well-known analogy one expresses the differential decay rate through a leptonic tensor $L_{\mu\nu}$ contracted with a hadronic tensor $W_{\mu\nu}$; the latter is decomposed into five Lorentz covariants with Lorentz invariant structure functions $w_i(q_0, q^2)$, where q_0 and $\sqrt{q^2}$ denote the energy and invariant mass of the lepton pair. Only two of those contribute, usually labelled w_1 and w_2, when lepton masses are neglected, and the semileptonic width is given by [48,49]

$$\Gamma_{SL} \equiv \Gamma(B \to lX_c) = \frac{G_F^2}{8\pi^3} \cdot |V(cb)|^2 \cdot \gamma \tag{275}$$

$$\gamma = \frac{1}{2\pi} \int_0^{q^2_{max}} dq^2 \int_{\sqrt{q^2}}^{q_{0,max}} dq_0 \sqrt{q_0^2 - q^2} \left[q^2 w_1(q_0, q^2) + \frac{q_0^2 - q^2}{3} w_2(q_0, q^2) \right] \tag{276}$$

where

$$q^2_{max} = (M_B - M_D)^2 \ , \ \ q_{0,max} = \frac{M_B^2 + q^2 - M_D^2}{2M_B} \tag{277}$$

In deep inelastic lepton-nucleon scattering QCD allows us to make two types of predictions: (i) If one manages to know – say from the data – what a given *moment* of a structure function is at a certain scale q_0^2, then one can predict what it should be at a higher scale q^2. (ii) Forming linear combinations of some moments of structure functions will project out the expectation value of a certain operator in the OPE. Symmetry considerations tell us the value of such matrix elements in certain instances, in which case we can predict the absolute magnitude of such moments. This is referred to as a *sum rule* like the Gross-Llewelyn-Smith sum rule or the Adler sum rule etc.

Both types of predictions can be made concerning heavy flavour decays as well. Information obtained on charm decays can be extrapolated to beauty decays to the degree that heavy quark expansions apply already at the charm scale (a proposition on which reasonable people can disagree). More powerful results can be deduced from the sum rules approach, in particular since the heavy quark symmetry yields much information about expectation values of various operators between H_Q states. The basic idea is the same underlying the QCD sum rules: one equates the integral over a transition rate evaluated on the quark-gluon level with the corresponding quantity *parametrized* in terms of hadronic quantities.

Some nontrivial results can be stated *without* actually evaluating moments. Let me illustrate this by citing three examples:

- Considering a semileptonic transition driven by the pseudoscalar weak current $J_5 = \int d^3x \{ci\gamma_5 b\}(x)$ one can deduce a sum rule at 'zero recoil' ($\vec{q} = 0$) for a structure function $w^{(5)}$:

$$\frac{1}{2\pi} \int_0^{\mu} d\epsilon w^{(5)}(\epsilon) = \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 \left(\mu_\pi^2(\mu) - \mu_G^2(\mu) \right) \tag{278}$$

where ϵ denotes the excitation energy above threshold, and $\mu_\pi^2(\mu)$ and $\mu_G^2(\mu)$ the expectation values of the kinetic and chromomagetic operator, respectively:

$$\mu_\pi^2(\mu) \equiv \frac{1}{2M_{H_Q}}\langle H_Q|\bar{Q}\vec{\pi}^2 Q|H_Q\rangle_{(\mu)} \ , \ \mu_G^2(\mu) \equiv \frac{1}{2M_{H_Q}}\langle H_Q|\bar{Q}\frac{i}{2}\sigma\cdot GQ|H_Q\rangle_{(\mu)} \tag{279}$$

With structure functions being nonnegative one deduces

$$\mu_\pi^2(\mu) \geq \mu_G^2(\mu) \tag{280}$$

with the normalization point μ provided by the cut-off in the integral on the left-hand side of Eq.(278). Eq.(280) forms an important element in several applications of the HQE.

- From a zero-recoil sum rule for the semileptonic transitions produced by the axial vector current $\bar{c}\gamma_i\gamma_5 b$ one infers in an analogous fashion a constraint on $|F_{D^*}(0)|$, the form factor for $B \to l\nu D^*$ at zero recoil. The result will be used later.

- Sum rules shed a considerable amount of light on the workings of quark-hadron duality to be discussed next.

Quark-Hadron Duality

The general notion that a quark-level based description should provide an approximation for a sufficiently inclusive hadronic transition goes back to the early days of the quark model, and it received a new impetus from the parton model. It did not represent a well-defined concept, though, and it was applied by different authors with considerable latitude. In a 1976 paper [45] Poggio et al. presented a more explicit discussion of how duality should work around the charm threshold in $e^+e^- \to had$: perturbative QCD allows to evaluate the total cross section in the Euclidean rather than the Minkowskian domain; the result is related to the observable cross section through a dispersion relation. This means that in general the physical cross section can be predicted only averaged – or 'smeared' – over some energy interval rather than energy-by-energy; it was estimated that this interval had to amount to 0.5 - 1 GeV. On the other hand, far away from any threshold the cross section would be a sufficiently smooth function such that smearing was no longer required and *local* duality applies.

Another milestone was reached in the mid 1980's when Shifman and Voloshin [46] first realized that under the special circumstance of heavy quark symmetry quark-hadron duality applies approximately even if only two channels dominate a transition – as it emerges for semileptonic B meson decays. This program was then pursued further and completed by Isgur and Wise [47] (see also [48]).

Heavy quark expansions have not solved the dynamical problems underlying quark-hadron duality, yet they have helped us in transforming it into a well-defined concept and have provided us with novel insights into its inner workings – at least for the dynamics of heavy flavours. In [50] the concept of *global* duality has been

introduced: it states that if QCD is to represent the theory of the strong interactions, the heavy flavour transition rate calculated in the Euclidean region – namely for the energy of the lepton pair q_0 purely imaginary – has to equal the dispersion integrals over the physical heavy flavour transition rates; for otherwise QCD had to generate a singularity in, say, the complex energy plane for which there were no physical counterpart!

This is however not the end of the story. For the dispersion integral extends over *all* physical heavy flavour transitions: decay processes as well as scattering and production processes! One has to argue then that the dispersion integral over the decay region alone is largely insensitive to the other physical singularities; those are referred to as the 'distant' cuts. This shows that duality in general cannot be more than an approximation the quality of which can depend on the specifics of the transition under study.

One has to keep in mind also that the OPE does not yield an expression that is convergent or even Borel summable. Evaluating the power expansion in the Euclidean domain does not enable us to determine those contributions that turn into *oscillating* terms in Minkowski space and introduce duality violations.

Soluble model field theories that exhibit quark confinement can be harnessed to obtain intriguing and non-trivial insights on the realization of duality and its limitations. This holds in particular for the t'Hooft model which is the $N_C \to \infty$ limit of 1+1 dimensional QCD. Based on *numerical* studies it has been claimed that at least in this model local duality is violated in the integrated widths of 'nonleptonic' decays (i) through the emergence of a $1/m_Q$ term *absent* in the OPE [51] and (ii) even more massively in explicitely flavour dependant processes like Weak Annihilation [52].

However it has been shown by us [53,54] through a careful *analytical* analysis that

- these claims are erroneous and
- their analysis flawed.

Since the spectrum of states in the t'Hooft model is known, one can calculate the inclusive width through the OPE on one hand and through a sum over the spectrum of the model on the other and compare the results. They are fully consistent with each other, i.e. *local duality does indeed hold in this model field theory to a high degree of accuracy and even the form and strength of duality violations can be estimated!*

The widths of weak decays sensitively depend on the phase space which is controlled by the masses of the states involved. It actually depends of a high power of these masses as exemplified by the simple parton model result

$$\Gamma(b \to cl\bar{\nu}) \propto G_F^2 m_b^5 f(x_c)\ , \ \ x_c = \frac{m_c^2}{m_b^2} \tag{281}$$

$$f(x) = 1 - 8x + 8x^3 - x^2 - 12x^2 \log x \tag{282}$$

It would then seem highly unlikely if a decay width given by *hadronic* masses could be approximated by a width expressed in terms of *quark* masses. Yet that is exactly what happens, and this apparent miracle is driven by the sum rules.

Let us consider the simple case of $b \to ul\bar{\nu}$ with $m_u = 0$ in a framework where the power of the b quark mass in Eq.(282) has been made a free parameter n in which one can expand transition rates [55]:

$$\Gamma(B \to lX_u) \propto |V(ub)|^2 G_F^2 m_b^5 \Longrightarrow \Gamma_n(B \to lX_u) \propto |V(ub)|^2 G_F^2 m_b^n \tag{283}$$

This can be achieved, for example, by having l extra scalar "leptons" emitted at the weak vertex; expressing the semileptonic width in terms of the structure functions then reads as follows:

$$\Gamma_n(B \to lX_u) = |V(ub)|^2 \frac{G_F^2}{8\pi^3} \cdot \gamma_n \tag{284}$$

$$\gamma_n = \frac{1}{2\pi}\int_0^{q^2_{max}} dq^2 (q^2)^l \int_{\sqrt{q^2}}^{q_{0,max}} dq_0 \sqrt{q_0^2 - q^2}\left[q^2 w_1(q_0,q^2) + \frac{q_0^2 - q^2}{3} w_2(q_0,q^2)\right] \tag{285}$$

with

$$n = 5 + 2l \tag{286}$$

and $l = 0$ in the real world. Using a more convenient set of kinematical variables

$$\epsilon = M_B - q_0 - \sqrt{M_D^2 + q_0^2 - q^2}\ , \ \ T = \sqrt{M_D^2 + q_0^2 - q^2} - M_D \tag{287}$$

one obtains

$$\gamma_n = \frac{1}{\pi}\int_0^{T_{max}} dT (T + M_D)\sqrt{T^2 + 2M_D T}\int_0^{\epsilon_{max}} d\epsilon (\Delta^2 - 2M_B T - 2\Delta\epsilon + 2T\epsilon + \epsilon^2)^l \cdot$$

$$\cdot \left[(\Delta^2 - 2M_B T - 2\Delta\epsilon + 2T\epsilon + \epsilon^2) w_1(T,\epsilon) + \frac{T^2 + 2M_D T}{3} w_2(T,\epsilon)\right] \tag{288}$$

with

$$\Delta = M_B - M_D\ ,\ T_{max} = \frac{\Delta^2}{2M_B}\ ,\ \epsilon_{max} = \Delta - T - \sqrt{T^2 + 2M_D T} \tag{289}$$

Expanding the integrand in Eq.(288) in ϵ we find through order ϵ

$$\gamma_n = \frac{1}{\pi}\int_0^{M_B/2} dT T^2 M_B^l (M_B - 2T)^l \left[M_B(M_B - 2T)\int_0^{M_B - 2T} d\epsilon w_1 - \right.$$

$$\left. 2(l+1)(M_B - T)\int_0^{M_B-2T} d\epsilon \epsilon w_1\right] +$$

$$+ \frac{1}{3\pi}\int_0^{M_B/2} dT T^4 M_B^{l-1}(M_B - 2T)^{l-1}\left[M_B(M_B - 2T)\int_0^{M_B-2T} d\epsilon w_2 - \right.$$

$$\left. 2l(M_B - T)\int_0^{M_B-2T} d\epsilon \epsilon w_2\right] \tag{290}$$

With the sum rules

$$\frac{1}{2\pi}\int d\epsilon w_1 = 1\ ,\quad \frac{1}{2\pi}\int d\epsilon\epsilon w_1 = M_B - m_b = \bar{\Lambda} \tag{291}$$

$$\frac{1}{2\pi}\int d\epsilon w_2 = \frac{2m_b}{T}\ ,\quad \frac{1}{2\pi}\int d\epsilon\epsilon w_2 = \frac{2m_b}{T}\bar{\Lambda} \tag{292}$$

integration over T yields

$$\gamma_n = \frac{M_B^{2l+5}}{2(l+4)(l+3)(l+2)}\left[1-(2l+5)\frac{\bar{\Lambda}}{M_B}\right] + \\ + \frac{m_b M_B^{2l+4}}{2(l+4)(l+3)(l+2)(l+1)}\left[1-(2l+4)\frac{\bar{\Lambda}}{M_B}\right] \tag{293}$$

Since

$$M_B\left(1-\frac{\bar{\Lambda}}{M_B}+\mathcal{O}(1/M_B^2)\right) = m_b \tag{294}$$

we see that the meson mass M_B is replaced by the quark mass m_b actually for *any* value of $n = 5+2l$ – due to the imposition of the sum rules! That means that the two aspects of long-distance dynamics, namely 'dressing up'

- the quark masses into hadron masses and
- quark operators into hadron operators,

actually cancel out as far as decay widths are concerned, and that this is implemented through the sum rules.

7.3 Basic Elements

Heavy Quark Masses

An internally consistent definition of the heavy quark mass is crucial for $1/m_Q$ expansions conceptually as well as for quantitative studies. While this remark is obvious in hindsight, the theoretical implications were at first not fully appreciated.

In QED it is very natural to adopt the pole mass for the electron, which is defined as the position of the pole in the electron Green function (actually the beginning of the cut, to be more precise): it is gauge invariant and it can be measured since it represents the mass of an isolated electron. For quarks the situation is qualitatively different because of confinement! Yet computational convenience suggested to use the pole mass for quarks as well: while not measurable, it is still gauge invariant and *perturbatively* infrared stable *order by order*. It thus constitutes a useful theoretical construct – as long as one addresses *purely perturbative* effects. Yet the pole mass is *not* infrared stable in *full* QCD – it exhibits a renormalon ambiguity:

$$\frac{\delta_{IR} m_Q^{pole}}{m_Q} \sim \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_Q}\right) \tag{295}$$

The origin of this *irreducible* uncertainty can be understood on physical grounds by considering the energy stored in the chromomagnetic field in a sphere of radius $R \gg 1/m_Q$ around a static colour source of mass m_Q:

$$\delta\mathcal{E}_{Coul}(R) \propto \int_{1/m_Q \leq |x| < R} d^3x \vec{E}^2_{Coul} \propto \text{const.} - \frac{\alpha_S(R)}{\pi}\frac{1}{R} \tag{296}$$

The definition of the pole mass amounts to setting $R \to \infty$; i.e., in evaluating the pole mass one undertakes to integrate the energy density associated with the colour source over *all space* assuming it has a Coulomb form as inferred from perturbation theory. Yet in the full theory the colour interaction becomes strong at distances approaching $R_0 \sim 1/\Lambda_{QCD}$, and the colour field can no longer be approximated by a $1/R$ field. Thus the long-distance or infrared region around and beyond R_0 cannot be included in a meaningful way; its contribution has to be viewed as an intrinsic uncertainty in the pole mass which is then estimated in accordance with Eq.(295) [x].

Why the pole mass is in principle inappropriate for heavy quark expansions can be read off from Eq.(295) directly: its uncertainty of order $1/m_Q$ would dominate the leading nonperturbative contributions of order $1/m_Q^2$ one works so hard to incorporate – in particular when entering through the high power m_Q^5! Instead one needs a *running* mass $m_Q(\mu)$ defined at a scale μ that shields it against the infrared dynamics. *In principle* any scale $\mu \gg \Lambda_{QCD}$ will do; yet *in practise* some provide a significantly more favourable computational environment than others. It can be shown that $\mu \sim 1$ GeV is an appropriate scale for these purposes whereas $\mu \simeq m_Q$ leads to higher order perturbative contributions that are artificially large [55].

From the measured masses of the charm and beauty hadrons one infers

$$m_b - m_c \simeq 3.50 + 40\,\text{MeV} \cdot \frac{\mu_\pi^2 - 0.5\,\text{GeV}^2}{0.1\,\text{GeV}^2} \pm 0.015\,\text{GeV} \tag{297}$$

An analysis of $e^+e^- \to$ *had* just above the threshold for open beauty production yields

$$m_b(1\,\text{GeV}) = 4.64\,\text{GeV} \pm 0.05\,\text{GeV} \tag{298}$$

These results could suffer from some potential theoretical uncertainties not stated in Eqs.(297, 298). In the future we will be able to extract m_b and m_c in a systematically independant way, namely from the *spectra* of semileptonic and radiative B decays.

Matrix Elements

Expanding the expectation value of the leading operator $\bar{Q}Q$ in powers of $1/m_Q$ yields

$$\frac{1}{2M_{P_Q}}\langle P_Q|\bar{Q}Q|P_Q\rangle = 1 - \frac{\mu_\pi^2}{2m_Q^2} + \frac{\mu_G^2}{2m_Q^2} + \mathcal{O}(1/m_Q^3) \tag{299}$$

where P_Q $[V_Q]$ denotes a pseudoscalar [vector] meson with quantum number Q.

[x] The reader can rest assured that Eq.(295) is derived in a more rigorous way.

The value of μ_G^2 for mesons is deduced from their hyperfine splitting:

$$\mu_G^2 = \frac{1}{2M_{P_Q}}\langle P_Q|\bar{Q}\frac{i}{2}\sigma\cdot GQ|P_Q\rangle = \frac{3}{4}(M_{V_Q}^2 - M_{P_Q}^2) \simeq \tag{300}$$

$$\simeq \frac{3}{4}\left(M_{B^*}^2 - M_B^2\right) \simeq 0.36\,\mathrm{GeV}\ . \tag{301}$$

For baryons one finds

$$\langle\Lambda_Q|\bar{Q}\frac{i}{2}\sigma\cdot GQ|\Lambda_Q\rangle \simeq \langle\Xi_Q|\bar{Q}\frac{i}{2}\sigma\cdot GQ|\Xi_Q\rangle \simeq 0 \neq \langle\Omega_Q|\bar{Q}\frac{i}{2}\sigma\cdot GQ|\Omega_Q\rangle\ , \tag{302}$$

since the light diquark system in Λ_Q and Ξ_Q carries spin zero, yet spin one for Ω_Q.

For the expectation value of the kinetic operator we have, as stressed above, a field theoretical inequality derived from the sum rules:

$$\mu_\pi^2 > \mu_G^2 \tag{303}$$

There is significant evidence that it cannot exceed this lower bound by a lot.

The expectation values of the dimension-six four-fermion operators, which provide the main motor driving differences in the decays widths of the various mesons of a given heavy flavour, cannot be deduced from first principles; their estimates suffer from still considerable uncertainties. Their size is usually calibrated by the so-called *factorization* or *vacuum saturation ansatz*. There is a lively debate in the literature between the advocates and the agnostics [57,58]. There is some recent evidence [59] from lattice simulations of QCD indicating that factorization apparently holds to better accuracy than anticipated by the agnostics. Two aspects that should not be in dispute (although it often goes unappreciated) are:

- The dynamical content of the factorization ansatz very significantly depends on the scale at which it is assumed; nonfactorizable contributions at one scale can become factorizable at a lower scale and vice versa!
- If factorization provides a valid approximation anywhere, it can be only at a low scale of around 1 GeV.

7.4 Lifetime Ratios

Predictions on the lifetime ratios among beauty hadrons were inferred from the Heavy Quark Expansions a few years ago well before data of sufficient accuracy were available [63,56,57]:

$$\frac{\tau(B^-)}{\tau(B_d)} \simeq 1 + 0.05\cdot\left(\frac{f_B}{200\,\mathrm{MeV}}\right)^2 \tag{304}$$

$$\frac{\bar{\tau}(B_s)}{\tau(B_d)} \simeq 1 \pm \mathcal{O}(0.01) \tag{305}$$

where $\bar{\tau}(B_s)$ denotes the average lifetime of the two B_s mass eigenstates.

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} \simeq 0.9 - 0.95 \tag{306}$$

More recently also the B_c lifetime was predicted using the B and D lifetimes as input [65,66]:

$$\tau(B_c) \simeq 0.5 \text{ psec} \tag{307}$$

Data from the LEP collaborations, CDF, SLD and CLEO yield as world averages

$$\frac{\tau(B^-)}{\tau(B_d)} = 1.07 \pm 0.03 \tag{308}$$

$$\frac{\bar{\tau}(B_s)}{\tau(B_d)} = 0.94 \pm 0.04 \tag{309}$$

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.79 \pm 0.05 \tag{310}$$

$$\tau(B_c) = 0.46 \pm 0.17 \text{ psec} \tag{311}$$

While the predictions on $\tau(B^-)/\tau(B_d)$, $\bar{\tau}(B_s)/\tau(B_d)$ and $\tau(B_c)$ are fully consistent with the data, the observed value of $\tau(\Lambda_b)$ is significantly lower than the predicted one. A few comments are in order to evaluate the situation:

- The prediction of Eq.(304) has been criticized theoretically as not sufficiently conservative [60]; yet a recent lattice simulation of QCD yielded

$$\frac{\tau(B^-)}{\tau(B_d)} = 1.03 \pm 0.02 \pm 0.03 \,, \tag{312}$$

  which is fully consistent with Eq.(304).

- Also Eq.(306) has been criticised theoretically [60]. Other studies, however, find [61]:

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 1 - \Delta \,, \quad \Delta \sim 0.03 - 0.12 \,, \tag{313}$$

  which again is fully consistent with the original prediction of Eq.(306), but hardly with the data. A recent careful reanalysis of four-quark operators [62] arrives at basically the same conclusion. No appealing fix for the prediction has been found within the framework of HQE.

- This discrepancy has prompted the radical ansatz that the weak widths of the various beauty hadrons scale with the fifth power of their hadronic mass rather than the b quark mass [64]:

$$\Gamma(H_b) \propto G_F^2 M_{H_b}^5 \tag{314}$$

  yielding [y]

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = \left(\frac{M_B}{M_{\Lambda_b}}\right)^5 \simeq 0.73 \tag{315}$$

[y] This feature was first noted in [63] well before there was any sign of a 'short' Λ_b lifetime.

This ansatz is quite radical in that it cannot be reconciled with the OPE approach: for it manages to drive down the Λ_b lifetime through a correction of order $1/m_b$ – and actually one with a large coefficient:

$$M^5_{\Lambda_b} = (m_b + \bar{\Lambda})^5 = m_b^5 \left(1 + 5\frac{\bar{\Lambda}}{m_b} + ...\right) \tag{316}$$

that is unequivocally anathema to the OPE! This is often referred to as a violation of local duality. On the phenomenological side one should note its prediction

$$\frac{\bar{\tau}(B_s)}{\tau(B_d)} \simeq \left(\frac{M_{B_d}}{M_{B_s}}\right)^5 \simeq 0.92 \; , \tag{317}$$

i.e., smaller than the HQE prediction, Eq.(305).

- I prefer to wait and see what the CDF/D0 data in run II will yield before seriously contemplating such drastic measures.

In passing I would like to add another comment: Predictions based on the *theoretical* framework of HQE implemented through the OPE are more unequivocal than those inferred from models with their ad-hoc assumptions as central elements. Then even a failure will teach us a meaningful, even if sad lesson.

7.5 Semileptonic Transitions

Semileptonic decays of beauty mesons represent a somewhat less complex dynamical scenario since factorization of the quark and the lepton bilinears is guaranteed to hold (although, in my judgement, this point is often exaggerated in the literature). Here I want to address one aspect of it only, although it is of central importance, namely the accurate extraction of the CKM parameters $|V(cb)|$ and $|V(ub)|$.

Total Semileptonic Widths

HQE yields for the total semileptonic width

$$\Gamma(B \to l X_c) = \frac{G_F^2 m_b^5 |V(cb)|^2}{192\pi^3} \times$$

$$\left[z_0(m_c^2/m_b^2) \cdot \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2}\right) - 2\left(1 - \frac{m_c^2}{m_b^2}\right)^4 \frac{\mu_G^2}{m_b^2} - \frac{2\alpha_S}{3\pi} \cdot z_0^{(1)}(m_c^2/m_b^2) + ... \right] \tag{318}$$

where the omitted terms are higher-order perturbative and/or power suppressed corrections; z_0 and $z_0^{(1)}$ are known phase space factors depending on m_c^2/m_b^2.

The leading nonperturbative corrections of order $1/m_b^2$ are small reducing the width by about 5 % and the direct impact of higher-order power corrections is estimated to be rather negligible. They exert, however, a very considerable indirect influence. For Eq.(318) makes it obvious that the choice of the heavy mass m_b is of crucial importance. As discussed before, usage of the pole mass would introduce

considerable intrinsic uncertainties that are actually larger than the leading non-perturbative contributions. Instead one has to use the running mass evaluated at a low scale μ chosen around 1 GeV.

Comparing this theoretical expresssion with the data leads to an extraction of $|V(cb)|$:

$$|V(cb)| = 0\ .0419\sqrt{\frac{BR(B\to lX_c)}{0.105}}\sqrt{\frac{1.55\,\mathrm{psec}}{\tau(B)}}\times \tag{319}$$

$$\times\left(1-0.012\cdot\frac{\mu_\pi^2-0.5\,\mathrm{GeV}^2}{0.1\,\mathrm{GeV}^2}\right)\left(1-0.01\cdot\frac{\delta m_b(\mu)}{50\,\mathrm{MeV}}\right)\times \tag{320}$$

$$\times\left(1+0.006\cdot\frac{\alpha_S^{\bar{\mathrm{MS}}}(1\,\mathrm{GeV}^2)-0.336}{0.02}\right)\left(1+0.007\cdot\frac{\bar\rho^3}{0.1\,\mathrm{GeV}^3}\right) \tag{321}$$

where $\bar\rho^3$ reflects the contributions from the $1/m_b^3$ terms. These detailed list of possible uncertainties can be analyzed – for details see [39] – yielding

$$|V(cb)| = 0\ .0419\sqrt{\frac{BR(B\to lX_c)}{0.105}}\sqrt{\frac{1.55\,\mathrm{psec}}{\tau(B)}}\times \tag{322}$$

$$\times\left(1-0.012\cdot\frac{\mu_\pi^2-0.5\,\mathrm{GeV}^2}{0.1\,\mathrm{GeV}^2}\right)\times \tag{323}$$

$$\times\ (1\pm 0.015_{pert}\pm 0.01_{m_b}\pm 0.012_{nonpert}) \tag{324}$$

I find it appropriate to combine various *theoretical* errors with all their correlations (and biases?) *linearly* rather than quadratically; thus I assign 5% as overall theoretical uncertainty to the extraction of $|V(cb)|$ from the total semileptonic width.

Some authors in the past have claimed a considerably larger theoretical uncertainty on this extraction. They expressed the theoretical width in terms of the *pole* masses to which they somehow assigned a 10% uncertainty. Using the BLM resummation they found the coefficients of the α_S^2 radiative corrections very large – between $\simeq -10$ and -20 for $b\to c$ and even $\simeq -30$ for $b\to u$ – reducing the overall width by rouhly 10% or so. If that were the whole story, one would have to argue that the theoretical uncertainties are considerably larger than stated above; it would actually be legitimate to be concerned whether the results of such a procedure could be trusted at all!

What was overlooked by those claims, however, is that those two uncertainties – the one in m_b and the one in the α_S expansion – are highly correlated to each other and thus combine to create a much smaller overall error. This can be best seen by evaluating the masses at a scale around 1 GeV, as advocated before.

The KM suppressed semileptonic width $B\to lX_u$ can be expressed in terms of $|V(ub)|$ with almost as good an accuracy [67]

$$|V(ub)| = 0\ .00465\sqrt{\frac{BR(B\to lX_u)}{0.002}}\sqrt{\frac{1.55\,\mathrm{psec}}{\tau(B)}}\times \tag{325}$$

$$\times\ (1\pm 0.025_{pert}\pm 0.03_{m_b}\pm 0.01_{nonpert}) \tag{326}$$

- since the energy release – the inverse of which constitutes the expansion parameter – is larger here than in $b \to c$,
- and the dependance on μ_π^2 is practically absent,
- whereas m_b is less well-known than the mass splitting $m_b - m_c$ which largely controls $b \to lX_u$.

Pilot studies by ALEPH and DELPHI exhibit a width for $B \to lX_u$ a bit below $2 \cdot 10^{-3}$ [68] suggesting

$$|V(ub)| \sim 4 \cdot 10^{-3} \tag{327}$$

somewhat larger, though consistent with what analyses of the exclusive modes $B \to l\nu\pi$ and $B \to l\nu\rho$ find where one has to invoke models for the hadronic form factors.

The data on $\Gamma(B \to l\nu X_u)$ will improve significantly with input from CLEO, BELLE and BABAR. One particularly useful discriminator between $\Gamma(B \to l\nu X_u)$ and $\Gamma(B \to l\nu X_c)$ will be provided by measuring the spectra of the hadronic recoil masses for which models exist that implement all known constraints from QCD [69,70,71].

$B \to l\nu D^*$

Introduction of the universal Isgur-Wise function was a crucial step in the evolution of heavy quark theory. It also provides great practical help in treating *exclusive* channels: it tells us that certain form factors – in particular the one for $B \to l\nu D^*$ – have to be normalized to unity in the infinite mass limit.

Analyzing the data on $B \to l\nu D^*$ and extrapolating them to the kinematical point of zero recoil yields

$$|F_{D^*}(0)V(cb)| = 0.0339 \pm 0.0014 \tag{328}$$

While $F_{D^*}(0) = 1$ holds asymptotically, it will receive power suppressed and perturbative corrections for finite quark masses:

$$F_{D^*}(0) = 1 + \mathcal{O}\left(\frac{\alpha_S}{\pi}\right) + \mathcal{O}\left(\frac{1}{m_c^2}\right) + \mathcal{O}\left(\frac{1}{m_c m_b}\right) + \mathcal{O}\left(\frac{1}{m_b^2}\right) \tag{329}$$

The absence of $1/m_Q$ corrections here noted in passing in [72] was cast into the form of a theorem by Luke [73].

Essential information on this form factor can be inferred from the sum rules for axialvector currents:

$$F_{D^*}(0) \simeq 0.91 - 0.013 \cdot \frac{\mu_\pi^2 - 0.5\,\mathrm{GeV}^2}{0.1\,\mathrm{GeV}^2} \pm 0.02_{excit} \pm 0.01_{pert} \pm 0.025_{1/m_Q^3} \tag{330}$$

where the number with the subscript *excit* represents the estimate for how much the excitations beyond D^* contribute to the sum rule. Putting everything together leads to

$$F_{D^*}(0) \simeq 0.91 \pm 0.06 \tag{331}$$

where I have again resisted the temptation to combine theoretical errors in quadrature!

From the data stated in Eq.(328) one then infers

$$|V(cb)|_{excl} = 0.0377 \pm 0.0016|_{exp} \pm 0.0025|_{theor} \tag{332}$$

The two determinations in Eqs.(324) and (332) are systematically very different both in their experimental and theoretical aspects. Nevertheless they are quite consistent with each other with the experimental and theoretical uncertainties being very similar. A few years ago it would have seemed quite preposterous to claim such small theoretical uncertainties!

Future Improvements

I am quite confident that the uncertainties on $|V(cb)|$ can be reduced from the present 5% level down to the 2% level in the foreseeable future.

$|V(ub)|$ (or $|V(ub)/V(cb)|$) is not known with an even remotely similar accuracy, and so far one has relied on models rather than QCD proper to extract it from data. Yet I expect that over the next ten years $|V(ub)|$ will be determined with a theoretical uncertainty below 10% . It will be important to obtain it from systematically different semileptonic distributions and processes; Heavy Quark Theory provides us with the indispensable tools for combining the various analyses in a coherent fashion.

This theoretical progress can embolden us to hope that in the end even $|V(td)|$ can be determined with good accuracy – say $\sim 10 \div 15\%$ – from $\Gamma(K^+ \to \pi^+ \nu \bar{\nu})$, $\Delta m(B_s)$ vs. $\Delta m(B_d)$ or $\Gamma(B \to \gamma \rho/\omega)$ vs. $\Gamma(B \to \gamma K^*)$ etc.

8 The Cathedral Builders' Paradigm

8.1 The Paradigm

The dynamical ingredients for numerous and multi-layered manifestations of CP and T violations do exist or are likely to exist. Accordingly one searches for them in many phenomena, namely in

- the neutron electric dipole moment probed with ultracold neutrons at ILL in Grenoble, France;
- the electric dipole moment of electrons studied through the dipole moment of atoms at Seattle, Berkeley and Amherst in the US;
- the transverse polarization of muons in $K^- \to \mu^- \bar{\nu} \pi^0$ at KEK in Japan;
- ϵ'/ϵ_K as obtained from K_L decays at FNAL and CERN and soon at DAΦNE in Italy;
- in decay distributions of hyperons at FNAL;
- likewise for τ leptons at CERN, the beauty factories and BES in Beijing;

- CP violation in the decays of charm hadrons produced at FNAL and the beauty factories;
- CP asymmetries in beauty decays at DESY, at the beauty factories at Cornell, SLAC and KEK, at the FNAL collider and ultimately at the LHC.

A quick glance at this list already makes it clear that frontline research on this topic is pursued at all high energy labs in the world – and then some; techniques from several different branches of physics – atomic, nuclear and high energy physics – are harnessed in this endeavour together with a wide range of set-ups. Lastly, experiments are performed at the lowest temperatures that can be realized on earth – ultracold neutrons – and at the highest – in collisions produced at the LHC. And all of that dedicated to one profound goal. At this point I can explain what I mean by the term "Cathedral Builders' Paradigm". The building of cathedrals required interregional collaborations, front line technology (for the period) from many different fields and commitment; it had to be based on solid foundations – and it took time. The analogy to the ways and needs of high energy physics are obvious – but it goes deeper than that. At first sight a cathedral looks like a very complicated and confusing structure with something here and something there. Yet further scrutiny reveals that a cathedral is more appropriately characterized as a complex rather than a complicated structure, one that is multi-faceted and multi-layered – with a coherent theme! One cannot (at least for first rate cathedrals) remove any of its elements without diluting (or even destroying) its technical soundness and intellectual message. Neither can one in our efforts to come to grips with CP violation!

8.2 Summary

- We know that CP symmetry is not exact in nature since $K_L \to \pi\pi$ proceeds and presumably because we exist, i.e. because the baryon number of the universe does *not* vanish.
- If the KM mechanism is a significant actor in $K_L \to \pi\pi$ transitions then there must be large CP asymmetries in the decays of beauty hadrons. In B^0 decays they are naturally measured in units of 10 %!
- Some of these asymmetries are predicted with high parametric reliability.
- New theoretical technologies will allow us to translate such parametric reliability into quantitative accuracy.
- Any significant difference between certain KM predictions for the asymmetries and the data reveals the intervention of New Physics. There will be no 'plausible deniability'.
- We can expect 10 years hence the theoretical uncertainties in some of the predictions to be reduced below 10 % .
- I find it likely that deviations from the KM predictions will show up on that level.

- Yet to exploit this discovery potential to the fullest one will have to harness the statistical muscle provided by beauty production at hadronic colliders.

8.3 Outlook

I want to start with a statement about the past: *The comprehensive study of kaon and hyperon physics has been instrumental in guiding us to the Standard Model.*

- The $\tau - \theta$ puzzle led to the realization that parity is not conserved in nature.

- The observation that the production rate exceeded the decay rate by many orders of magnitude – this was the origin of the name 'strange particles' – was explained through postulating a new quantum number – 'strangeness' – conserved by the strong, though not the weak forces. This was the beginning of the second quark family.

- The absence of flavour-changing neutral currents was incorporated through the introduction of the quantum number 'charm', which completed the second quark family.

- CP violation finally led to postulating yet another, the third family.

All of these elements which are now essential pillars of the Standard Model were New Physics at *that* time!

I take this historical precedent as clue that a detailed, comprehensive and thus neccessarily long-term program on beauty physics will lead to a new paradigm, a *new* Standard Model!

CP violation is a fundamental as well as mysterious phenomenon that we have not understood yet. This is not surprising: after all according to the KM mechanism CP violation enters through the quark mass matrices; it thus relates it to three central mysteries of the Standard Model:

- How are fermion masses generated? [z]

- Why is there a family structure?

- Why are there three families rather than one?

In my judgement it would be unrealistic to expect that these questions can be answered through pure thinking. I strongly believe we have to appeal to nature through experimental efforts to provide us with more pieces that are surely missing in the puzzle. CP studies are essential in obtaining the full dynamical information contained in the mass matrices or – in the language of v. Eichendorff's poem quoted in the beginning, "to find the magic word" that will decode nature's message for us.

Considerable progress has been made in theoretical engineering and developing a comprehensive CP phenomenology from which I conclude:

[z] Or more generally: how are masses produced in general? For in alternative models CP violation enters through the mass matrices for gauge bosons and/or Higgs bosons.

- B decays constitute an almost ideal, certainly optimal and unique lab. Personally I believe that even if no deviation from the KM predictions were uncovered, we would find that the KM parameters, in particular the angles of the KM triangle, carry special values that would give us clues about New Physics. Some very interesting theoretical work is being done about how GUT dynamics in particular of the SUSY (or Supergravity) variety operating at very high scales would shape the observable KM parameters.

- A comprehensive analysis of charm decays with special emphasis on $D^0 - \bar{D}^0$ oscillations and CP violation is a moral imperative! Likewise for τ leptons.

- A vigorous research program must be pursued for light fermion systems, namely in the decays of kaons and hyperons and in electric dipole moments. After all it is conceivable of course that no CP asymmetries are found in B decays on a measurable level. Then we would know that the KM ansatz is *not* a significant actor in $K_L \to \pi\pi$, that New Physics drives it – but what kind of New Physics would it be? Furthermore even if large CP asymmetries were found in B decays, it could happen that the signals of New Physics are obscured by the large 'KM background'. This would not be the case if electric dipole moments were found or a transverse polarization of muons in $K_{\mu 3}$ decays.

- Close feedback between experiment and theory will be essential.

As the final summary: insights about Nature's Grand Design that can be obtained from a comprehensive and detailed program of CP studies

- are of fundamental importance,

- cannot be obtained any other way and

- cannot become obsolete!

Acknowledgments: I am deeply grateful to my collaborators A. Sanda, M. Shifman, N. Uraltsev and A. Vainshtein for their generosity in sharing their insights with me over so many years. I also would like to thank the organizers of this school – in particular Profs. J.B. Choi and D.S. Hwang – for their invitation to lecture in such a sophisticated and ancient metropolis, for making my stay so truly enjoyable and introducing me to Korea's fascinating history and culture. This work was supported by the National Science Foundation under the grant numbers PHY 92-13313 and PHY 96-05080.

1. For an updated review see: A.D. Dolgov, in: Proceedings of the XXVth ITEP Winterschool of Physics, Feb. 18 - 27, Moscow, Russia.
2. H.A. Kramers, *Proc. Acad. Sci. Amsterdam* **33** (1930) 959; see also: F.J. Dyson, *J. Math. Phys.* **3** (1962) 140; A. Messiah, *Quantum Mechanics, Vol. II*, North-Holland Publishing Co., Amsterdam (1965), p. 675.

3. I.I. Bigi, ‘CP Violation – An Essential Mystery in Nature’s Grand Design’, lectures given at the International School of Physics ”Enrico Fermi”, Course ‘Heavy Flavour Physics: A Probe of Nature’s Grand Design’, to appear in the Proceedings.
4. M.L. Good, *Phys. Rev.* **121** (1961) 311.
5. T.D. Lee, *Particle Physics and Introduction to Field Theory*, harwood academic publishers, New York (1981).
6. a very detailed discussion can be found in: I.I. Bigi, A.I. Sanda, *CP Violation*, Cambridge University Press, to appear.
7. P. Bloch, in: ”Heavy Quarks at Fixed Target”, L. Köpke (ed.), Frascati Physics Series, Vol. VII, 1997.
8. I have learnt this notation – using $\Phi(\Delta S = 1, 2)$ rather than ϵ_K and ϵ' – from B. Winstein.
9. L.M. Sehgal and M. Wanninger, *Phys. Rev.* **D46** (1992) 1035; *Phys. Rev.* **D46** (1992) 5209 (E); see also the earlier papers: A.D. Dolgov and L.A. Ponomarev, *Sov. J. Nucl. Phys.* **4** (1967) 262; D.P. Majumdar and J. Smith; *Phys.Rev.* **187** (1969) 2039.
10. J.K. Elwood, M.B. Wise and M.J. Savage, *Phys.Rev.* **D52** (1995) 5095; **D53** (1996) 2855(E); J.K. Elwood, M.B. Wise, M.J. Savage and J.M. Walden, *Phys.Rev.* **D53** (1996) 4078.
11. P. Heiliger and L.M. Sehgal, *Phys. Rev.* **D48** (1993) 4146.
12. M. Gell-Mann, A. Pais, *Phys. Rev.* **97** (1955) 1387.
13. J.H. Christenson, J.W. Cronin, V.L. Fitch, R. Turlay, *Phys. Rev. Lett.* **13** (1964) 138.
14. L. B. Okun, *Weak Interactions of Elementary Particles*, Pergamon, 1965; the Russian original had appeared in 1963, i.e. clearly *before* the discovery of CP violation.
15. L. Wolfenstein, *Phys. Rev. Lett.* **13** (1964) 562.
16. M. Kobayashi, T. Maskawa, *Prog. Theor. Phys.* **49** (1973) 652.
17. T. Inami, C.S. Lim, *Prog. Theor. Phys.* **65** (1981) 297.
18. G. Buchalla, A.J. Buras, M.E. Lautenbacher, *Rev. Mod. Phys.* **68** (1996) 1125.
19. A.J. Buras, M. Jamin, M.E. Lautenbacher, *Phys. Lett.* **B389** (1996) 749.
20. M. Ciuchini, preprint hep-ph/9701278, Invited Talk at the 4th KEK Topical Conference on ”Flavor Physics”, KEK, Japan, Oct. 29 - 31, 1996.
21. P. Franzini, in: Les Rencontres de Physique de la Vallee d’Aoste, Results and Perspectives in Particle Physics, M. Greco (ed.), Editions Frontieres, 1991.
22. S.R. Blatt et al., *Phys. Rev.***D 27** (1983) 1056.
23. R. Garisto, G. Kane, *Phys. Rev.***D 44** (1991) 2038.
24. A very nice review can be found in: W. Bernreuther, M. Suzuki, *Rev. Mod. Physics* **63** (1991) 313.
25. I.I. Bigi, Y. Dokshitzer, V. Khoze, J. Kühn, P. Zerwas, *Phys. Lett.* **B181** (1986) 157.
26. A. Carter, A.I. Sanda, *Phys. Rev.***D 23** (1981) 1567.
27. I.I. Bigi, A.I. Sanda, *Nucl. Phys.***B 193** (1981) 85.
28. M. Bander, D. Silverman, A. Soni, *Phys. Rev. Lett.***43** (1979) 242.

29. I. I.Bigi, V.A. Khoze, N.G. Uraltsev, A.I. Sanda, in: "CP Violation", C. Jarlskog (ed.), World Scientific, 1989.
30. I. Dunietz, *Phys. Rev.* **D 52** (1995) 3048.
31. K. Akerstaff *et al.*, Eur.Phys.J.**C5** 379,(1998).
32. F. Abe *et al.*, *Phys. Rev. Lett.* **81** (1998) 5513.
33. M. Gronau, in: "Heavy Quarks at Fixed Target", L. Köpke (ed.), Frascati Physics Series, Vol. VII, 1997.
34. M. Gronau, D. Wyler, *Phys. Lett.* **B 265** (1991) 172.
35. I.I. Bigi, A. Sanda, *Phys. Lett.* **B 221** (1988) 213.
36. I.I. Bigi, F. Gabbiani, *Nucl. Phys.* **B 352** (1991) 309.
37. M. Voloshin, M. Shifman, N. Uraltsev, V. Khoze, *Sov. J. Nucl. Phys.* **46** (1987) 112.
38. An excellent introduction into the state of the art for the novice can be found in: N. Uraltsev, "Heavy Quark Expansion in Beauty and its Decay", lectures given at the International School of Physics "Enrico Fermi", Course 'Heavy Flavour Physics: A Probe of Nature's Grand Design', to appear in the Proceedings.
39. A more concise review with a critical overview of the literature is given in: I.I. Bigi, M.Shifman, N. Uraltsev, *Annu. Rev. Nucl. Part. Sci.* **47** (1997) 591, with references to earlier work.
40. I.I. Bigi, A.I. Sanda, *Phys. Lett.* **B 171** (1986) 320.
41. I.I. Bigi, in: Proceed. of the XXIII International Conf. on High Energy Physics, Berkeley, USA, S.C. Loken (ed.), World Scientific, 1986, p. 857.
42. Y. Nir, in: Proceed. of the 6th International Symposium on Heavy Flavour Physics, Pisa, Italy, F. Costantini and M. Giorgi (eds.), 1995, p. 991.
43. I.I. Bigi, H. Yamamoto, *Phys. Lett.* **B 349** (1995) 363.
44. B. Chibisov, R. Dikeman, M. Shifman and N. Uraltsev, *Int. J. Mod. Phys.* **A12** (1997) 2075.
45. E. Poggio, H. Quinn, S. Weinberg, *Phys. Rev.* **D13** (1976) 1958.
46. M. Voloshin, M. Shifman, *Sov. J. Nucl. Phys.* **41** (1985) 120.
47. N. Isgur and M. Wise, *Phys. Lett.* **B232** (1989) 113; *Phys. Lett.* **B237** (1990) 527.
48. J. Chay, H. Georgi, B. Grinstein, *Phys. Lett.* **B247** (1990) 399.
49. B. Blok, L. Koyrakh, M. Shifman and A. Vainshtein, *Phys. Rev.* **D 49** (1994) 3356.
50. I. Bigi, M. Shifman, N. Uraltsev, A. Vainshtein, *Phys. Rev.* **D52** (1995) 196.
51. B. Grinstein and R. Lebed, *Phys. Rev.* **D 57** (1998) 1366.
52. B. Grinstein and R. Lebed, preprint hep-ph/9805404.
53. I. Bigi, M. Shifman, N. Uraltsev and A. Vainshtein, preprint hep-ph/9805241, accept. for publication in *Phys. Rev.* **D**.
54. I. Bigi and N. Uraltsev, preprint UND-HEP-99-BIG01.
55. I. Bigi, M. Shifman, N. Uraltsev, A. Vainshtein, *Phys. Rev.* **D 56** (1997) 4017.
56. I.I. Bigi, B. Blok, M. Shifman, N. Uraltsev and A. Vainshtein, in: 'B Decays', S. Stone (ed.), World Scientific, 1994, Revised 2nd Ed., p. 132.
57. B. Bellini, I. Bigi and P. Dornan, *Phys. Rep.* **289** (1997) 1.

58. M. Neubert, preprint hep-ph/9707217.
59. M. Di Pierro and C. Sachrajda, *Nucl. Phys.* **B 534** (1998) 373.
60. M.Neubert and C. Sachrajda, *Nucl. Phys.* **B 483** (1997) 339.
61. N. Uraltsev, *Phys. Lett.* **B 376** (1996) 303; J.L. Rosner, *Phys. Lett.* **B 379** (1996) 267.
62. D. Pirjol and N. Uraltsev, *Phys. Rev.* **D 59** (1999), hep-ph/9805488.
63. I. Bigi and N. Uraltsev, *Phys. Lett.* **B 280** (1992) 271.
64. G. Altarelli, G. Martinelli, S. Petrarca and F. Rapuano, *Phys. Lett.* **B 382** (1996) 409.
65. I.I. Bigi, *Nucl. Instr. & Methods in Phys. Res.* **A 351** (1994) 240; *Phys. Lett.* **B 371** (1996) 105.
66. M. Beneke and G. Buchalla, *Phys. Rev.* **D 53** (1996) 4991.
67. N. Uraltsev, *Int. Jour. Mod. Phys.* **A 11** (1996) 515.
68. A. Stocchi, talk given at HQ98, International Workshop on Heavy Quarks at Fixed Target, Fermilab, Oct. 10 - 12. Oct. 1998.
69. R. Dikeman and N. Uraltsev, *Nucl. Phys.* **B 509** (1998) 378.
70. I. Bigi, R. Dikeman and N. Uraltsev, *Eur. Phys. J.* **C 4** (1998) 453.
71. A. Falk, Z. Ligeti and M. Wise, *Phys. Lett.* **B 406** (1997) 225.
72. M. Voloshin, M. Shifman, *Sov. J. Nucl. Phys.* **47** (1988) 511.
73. M. Luke, *Phys. Lett.* **B 252** (1990) 447.